

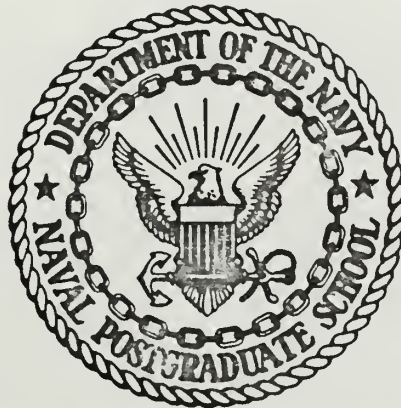
LONG WAVE STUDY OF MONTEREY BAY

by

Thomas John Lynch



# United States Naval Postgraduate School



## THESIS

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September 1970

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Long Wave Study of Monterey Bay

by

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requirements for the degree of

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ABSTRACT

Monterey Bay, on the west coast of the United States, is unique in that it is a large, symmetric, semi-elliptical bay divided north and south by the deep Monterey Canyon. The effect of the canyon on seiching within the bay and on long wave oscillations within the bay was studied by analyzing synchronized wave records at each end of the bay. Power spectra and cross spectra calculated for five periods selected from six months continuous data indicate the Monterey Canyon has a profound effect on the bay's oscillating characteristics. The canyon appears to act as an impedance barrier dividing the bay into two independent oscillating basins each having recurring long-period waves which persist during significant long-wave activity.





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## I. INTRODUCTION

Monterey Bay (Figure 1) lies on the west coast of the United States, located about sixty nautical miles south of San Francisco. The Bay is a large semi-elliptical bay which has some topographical features which are unique. A deep trough of the North Pacific basin, bounded on the north by the Mendocino Seascarp and on the south by the Murray Seascarp, approaches closer to the coast at Monterey Bay than at any other point along the North American coastline. As a result, long waves propagated across the Pacific Ocean in an easterly direction may be somewhat contained. The continental shelf within the bay extends out to approximately the 600 foot depth contour and is cut by the deep Monterey Canyon which has a volume of over 50 cubic miles. Monterey Canyon will be seen to exert significant influences on the oscillating characteristics of the Bay.

There have been several studies concerned with the effects of long-period waves in Monterey Bay (a long-period wave for this study is defined as a wave having a period in excess of 1 minute), but these studies have not been able to define the causes of the long wave activity and have left many areas of inquiry. Hudson [1947] proposed a surge model of Monterey Harbor for the U. S. Army Corps of Engineers. He used six months data (October 1946 - April 1947) from three electronically synchronized sea-surface recorders. These results are tabulated in Table I. The presence of the long-period waves was observed. However, the cause of the surge mechanism was unknown.

Wilson [1965] conducted a long-wave study of Monterey Bay for the U. S. Army Corps of Engineers to determine the feasibility of



construction of an engineering model of the surge phenomenon that occurred at various times within Monterey Bay. Statistical data were collected from sensors located at Monterey Harbor, and Santa Cruz (Figure 1). Three long-period wave recorders were also installed within Monterey Harbor and were in continuous operation from October 1963 to April 1964. Two were arranged so that tides and sea-swell were filtered out, and a third recorded swell approaching the harbor. Wilson concluded that the surge within Monterey Harbor was not a result of incoming swell as no correlation was found to exist between long-period waves within Monterey Harbor and the incoming sea-swell. The Santa Cruz sensor was designed so that it functioned as a long-wave recorder. The recorder operated continuously from October 1963 to February 1964. Wilson's results for Monterey Harbor are summarized in Tables II and III.

Wilson performed a residuation analyses of different records from sensors located around the bay in order to determine a local evaluation of the oscillations within the bay. Residuation analyses is accomplished by successively subtracting "apparent" periods from the wave record until a smooth trace remains. The subjectivity of which this method is accomplished can lead to fictitious results. These results are listed in Table IV. Table V is a synopsis of spectrum analyses for three days record of the Monterey sensors. Wilson concluded, based on Tables IV and V along with calculated modes of oscillation, Figures 24-29, that Monterey Canyon functions as an impedance between north and south portions of the bay, and therefore, free oscillations are to a large extent uncoupled. Wilson further states that the effect of any sharp discontinuity in submarine topography, equivalent to the edge of a



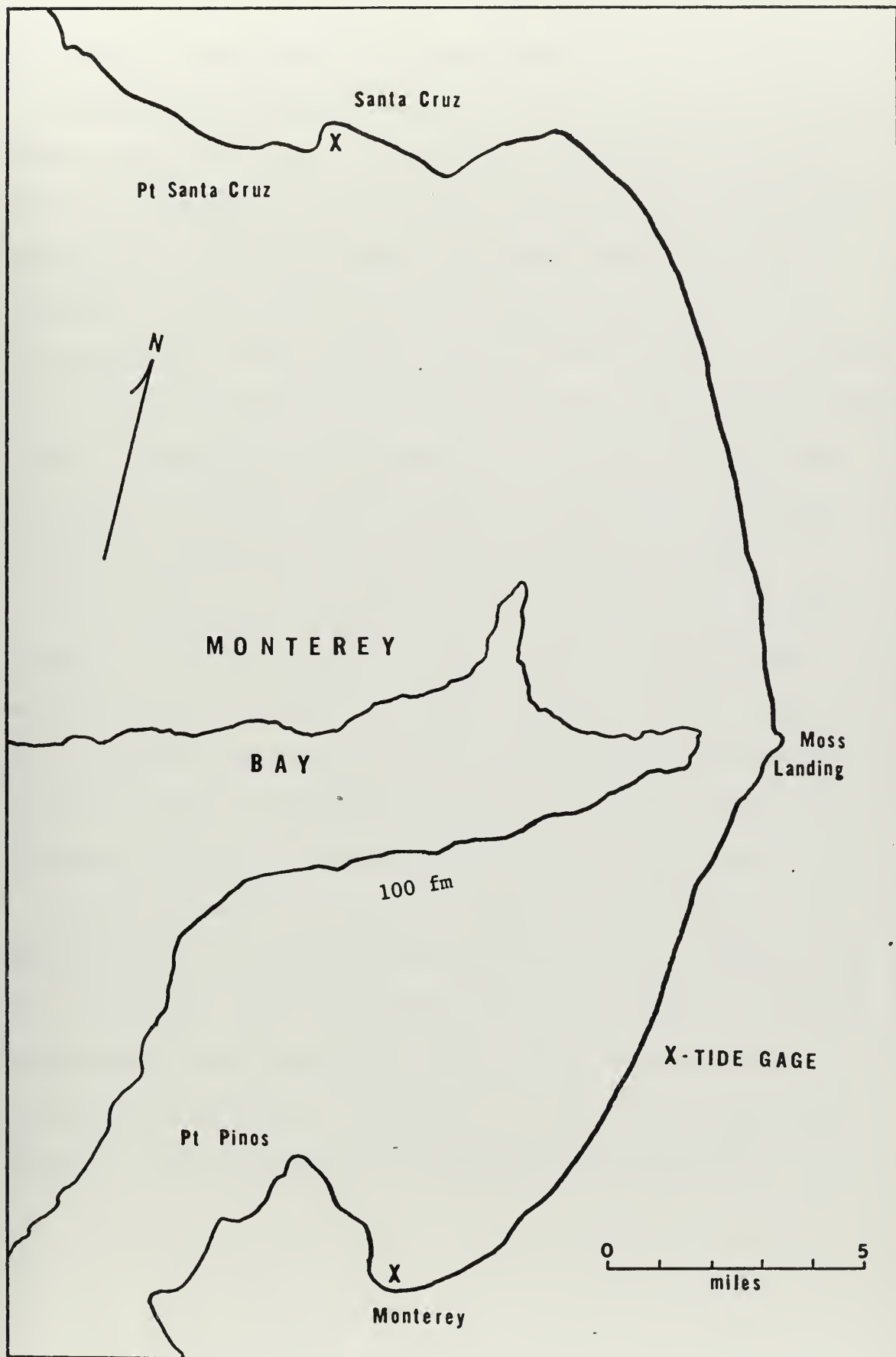


FIGURE 1



continental shelf, is to serve as a nodal position for any shelf oscillation to which the shelf is susceptible.

Robinson [1969] analyzed long-wave activity in Monterey Bay on 23 January and 20 April 1969. Tide recorders were located at Monterey Wharf #2 and Santa Cruz Municipal Wharf, however, the records were not synchronized. Thus, cross spectra and phase information was not obtainable. Robinson concluded that Monterey Canyon affected the seiching motions within the bay, however, it did not appear to divide the bay into two independent oscillating basins, as concluded by Wilson [1965]. Similar periods and amplitudes were found at Monterey and Santa Cruz concurrently, but correlation between the two locations could not be computed due to non-synchronization of records.

In the present study simultaneous tidal records from tide gages located at Monterey Wharf #2 and Santa Cruz Municipal Wharf were analyzed during periods of significant long-wave activity in Monterey Bay. Continuous analog records were maintained from 3 November 1969 to 30 April 1970 and five days (6 November, 3 December, 6 December, 14 January, 20 January) were selected for analysis and comparison.

The objectives were to perform an overall long-wave study of Monterey Bay, and to examine various modes of free oscillation of the bay to determine if in fact seiching does exist in the bay. This was accomplished through examination of the individual power spectra at Monterey and Santa Cruz, computation of the cross spectra and determination of phase differences of long-period waves present.





TABLE I

Long-Period Waves in Monterey Harbor (after Hudson, 1949)

PERIOD (min)	AVERAGE HEIGHT (ft)	PERCENTAGE OF TIME PRESENT
1-2	0.4	20
2-4	0.5	30
4-15	not given	15

TABLE II

Marine Advisor's Data for Monterey Harbor (after Wilson, 1965)

SENSOR	PERIOD (min)	AVERAGE HEIGHT (ft)	PERCENT OF TIME PRESENT
1	1.7-14	0.1-2.5	0-50
2	1-14	0.1-3.0	0-55

TABLE III

Marine Advisor's Data for Santa Cruz Harbor (after Wilson, 1965)

PERIOD (min)	AVERAGE HEIGHT (ft)	PERCENT OF TIME PRESENT
1-2.3	}	0-18
2.3-4		20-70
10-14		0-25



DATE & TIME	SENSOR LOCATION	SEQUENCE OF APPARENT PERIODS - mins. or secs. (s)														
		60	36.3	22.6		13.0	8.8		5.9							
1400, Mar. 29, 1964 -0400, Mar. 30, 1964	Monterey Tide Gage	60	36.3	22.6		13.0	8.8		5.9							
March 29, 1964 0000-0900	Monterey Tide Gage	60	35.7 32.3	19.6 17.3		11.1	9.3		5.5							
Feb. 10, 1964	Monterey Sensor No. 1				17.1 (?)		8.6 (?)			3.8	2.5		1.00	42-45s		
	Monterey Sensor No. 2				17.2				5.7	4.1		1.95	1.20			
	Monterey Sensor No. 3										1.7			44.8s	23.4s	15.0s 9.3s 6.9s
March 12, 1965 0850-1000 1000-1100 1100-1200	Outside Municipal Wharf No. 2					13.5 12.1	8.4 8.4		6.1	4.6 4.4 4.2	2.3 2.8 2.9		1.30 1.32 1.35	0.98 1.00 0.90	42s 36s 24.0s 24.0s	
May 27, 1947 1148-1214	Moss Landing									3.9 4.1	2.5 2.2	1.96	1.25	50.1s 32s 31s		
June 2, 1947	Moss Landing									4.6	2.3		1.4	47s	11.3s	7.8s
Nov. 24, 1964	Santa Cruz (1)	66	30	23.2		13.6 11.0		6.3								
	Santa Cruz (2)	86 (?)	28.5			13.0				3.8						

TABLE IV  
Periods of Oscillation from Residuation Analysis (From Wilson, 1965)



Date	Monterey Sensor No.	Sequence of Periods (mins.)												
			22.2		13.3	9.5	7.4		4.9	4.0		2.5	2.2	1.7
Case A Feb. 11, 1964	1													
	2		22.2		13.3	9.5		6.1	5.0				2.2 2.0	1.8
	3		22.2		13.3	9.5			5.3	4.3	3.3 2.9	2.6	2.1 2.0	
Case B March 28, 1964 (tsunami)	1	33.3					8.3	5.8		4.3	3.2	2.4	2.0	
	2	33.3		16.7			8.3		5.3	4.4	3.3	2.5	2.3	
	3	33.3		16.7		9.5		6.1		4.3	3.5	2.4	2.1	
Case C April 12, 1964	1		22.2		13.3	9.5	7.4			3.9		2.6	2.2 2.0	1.9
	2		22.2		13.3			6.7	5.1	4.0	3.0	2.5	2.2	1.9
	3		22.2		13.3	9.5	7.4	6.1	5.1	4.0	3.2	2.4	2.2	1.9

TABLE V  
Periods of Oscillation from Spectral Analysis (From Wilson, 1965)



## II. INSTRUMENTATION

### A. MONTEREY

The data for Monterey was obtained using a standard Coast and Geodetic Survey automatic tide gage. (Manual of Tide Observations, 1965), located on Monterey Municipal Wharf #2, Monterey Harbor. The tide gage is maintained daily by NPS personnel.

The Monterey tide gage senses changes in water level by means of a float/pulley arrangement, which is entirely mechanical. The recording drum is advanced by a clock mechanism. The drum speed is designed for 1 in/hr, however, the hourly feed was measured to be 1.03 in/hr. The instrument is time checked daily with time marks being accurate within  $\pm 5$  seconds. The marigram is recorded in rectilinear coordinates on plain white paper. A stilling well, which is a 12 inch diameter steel pipe with a 1 inch orifice in the bottom, serves as low pass filter for the majority of wind waves (periods of one minute and below).

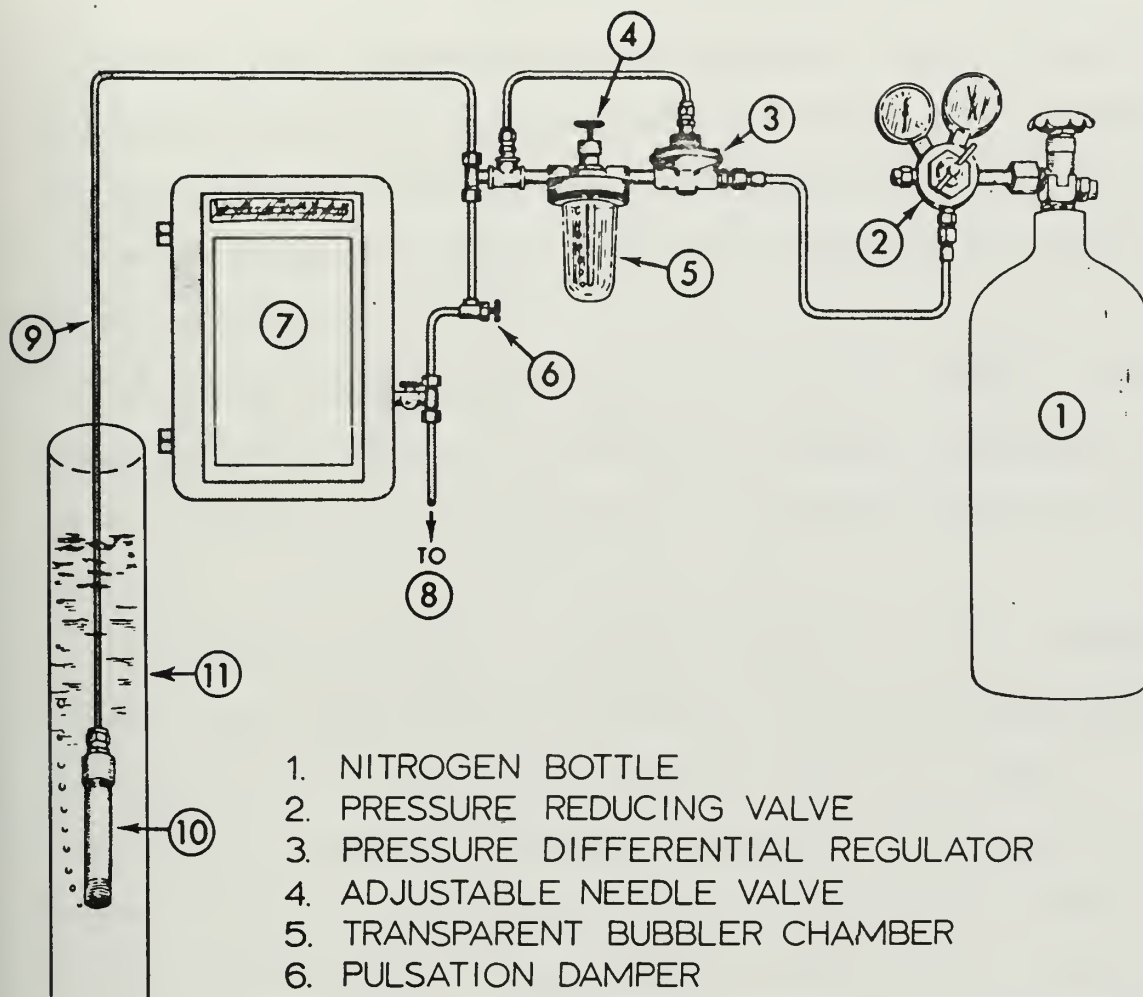
### B. SANTA CRUZ

The Santa Cruz data were obtained with a Bristol Model 28 gas-purging pressure (bubbler), portable tide gage, (Figure 2), located on Santa Cruz Municipal Wharf. This instrument senses changes in water level by means of a nitrogen-filled tube which is connected to a bellows system (Manual of Tide Observation, 1965).

A bubbler orifice chamber was connected to the end of the sensing tube to reduce sensitivity to short period wave action. There is also a bellows inlet needle valve which could be throttled to filter additional wave action from the record.







1. NITROGEN BOTTLE
2. PRESSURE REDUCING VALVE
3. PRESSURE DIFFERENTIAL REGULATOR
4. ADJUSTABLE NEEDLE VALVE
5. TRANSPARENT BUBBLER CHAMBER
6. PULSATION DAMPER
7. STRIP CHART RECORDER WITH TRANSDUCER
8. TELEMETERING PRESSURE TRANSMITTER
9. TUBING
10. BUBBLER ORIFICE CHAMBER
11. STILLING WELL

FIGURE 2 THE BUBBLER TIDE GAGE.



The pressure reduction valve is regulated to allow a constant pressure greater than that due to the maximum head anticipated over the orifice. The pressure differential regulator together with the shunt line provide for a constant pressure difference of about 3 pounds per square inch across the flow regulator. Thus, the rate of flow can be adjusted to a desired constant value and is not dependent on the tide stage. Flow is through a silicone oil-filled bowl so that it can be monitored visually. A rate of 30-60 bubbles per minute was used for this study.

Advantages of this scheme are that the sensitive elements of the gage need not be designed to withstand the underwater environment; and fluctuations in barometric pressure are not directly reflected on the record.

A significant portion of the system capacity is at the submerged orifice, thus providing some high frequency filtering and protecting the small diameter supply line and orifice from marine fouling.

A Bristol Company clock recorder provides an analog record. The record is on a 6 inch (15.2 cm) wide strip chart and is recorded in curvilinear coordinates. Paper advance is adjustable and 6 in/hr was utilized. The clock drive has an eight-day spring, and the record is sufficient for about seven days. Manufacturer's claims are that hysteresis and/or non-linearity are limited to 1% of the full-scale value. For sea water use, the instrument is calibrated to correspond to a specific gravity of 1.025.

In addition to the above high-frequency filtering mechanisms, a stilling well was designed and installed to attain the desired filtering of high frequency and waves [Robinson, 1966]. The well was



constructed of a 20 foot section of polyvinyl chloride (pvc) 6 inch inside diameter pipe. It was capped on 16, 1/4 inch inside diameter holes drilled in the side. Copper sleeves were inserted to eliminate fouling. The 16 holes provided the capability of increasing the orifice from a 1/4 inch to a 1 inch diameter opening.

The response characteristics of the well were determined theoretically for two orifice sizes and three different wave frequencies using the equation for the rate of water rise in a well [Robinson, 1969].

$$dh_i/dt = 0.6 a/A \sqrt{2g(h_0 - h_i)}$$

where,

$a$  = orifice area

$A$  = well area

0.6 = empirical orifice flow coefficient

$g$  = acceleration due to gravity

$h_0$  = water height outside the well, i.e., the forcing function

$h_i$  = water height inside the well.

The forcing function,  $h_0$ , was chosen to be a simple sine function of unit amplitude, and of frequency equal to the wave frequency of interest. The initial conditions were  $h_i = 0$  at  $t = 0$ . The results are summarized in Table VI.

The response characteristics for the Monterey stilling well were not calculated since the orifice area to well ratio is larger than the Santa Cruz well, providing response characteristics better than those of the Santa Cruz. A 1/4 inch orifice was used in the Santa Cruz well.



TABLE VI

Computed Response Characteristics of Santa Cruz Stilling Well  
(from Robinson, 1965)

PERIOD	ORFICE DIAMETER (in.)	PHASE LAG (deg)	AMPLITUDE REDUCTION (%)
20 sec	0.25	180	95
20 sec	1.00	72	45
60 sec	0.25	75	91
60 sec	1.00	30	5
25 min	0.25	6	1
25 min	1.00	0	0





### III. ANALYSIS PROCEDURE

#### A. MONTEREY

The Monterey tidal records were digitized at 100 points per inch giving a sampling interval of 34.95 seconds. The discrete Fast Fourier Transform utilized requires  $2 \times 2^m$  data points, and for the records analyzed, 2048 data points were used which gave a record length of 71545 seconds, or approximately 20 hours. In order to remove the tidal influence and reduce "leakage", the records were high pass filtered by fitting a least-squares polynomial curve to the raw data and the curve was then subtracted from each ordinate point (Appendix C). An example of the fit to sample data points is shown in Figure 3. It was found that the 6th-degree polynomial provided the best fit for this particular tidal cycle.

Once the tidal effects were removed, the data is subjected to the Fast Fourier Transform to obtain the energy density and phase estimates as a function of frequency. The IBM/360 standard subroutines HARM and RHARM were used to calculate the Fourier coefficients, from which the one-dimensional spectra are derived. This method and procedure is explained in Appendix B.

Subroutine RHARM gives the raw Fourier's coefficients

$$A_0/2, B_0 = 0, A_1, B_1, A_2, B_2, \dots, A_N/2, B_N = 0$$

which are then combined as

$$\frac{A_j}{2} + \frac{iB_j}{2} \quad j = 1, 2, \dots, N$$

to give the one-sided spectral estimate



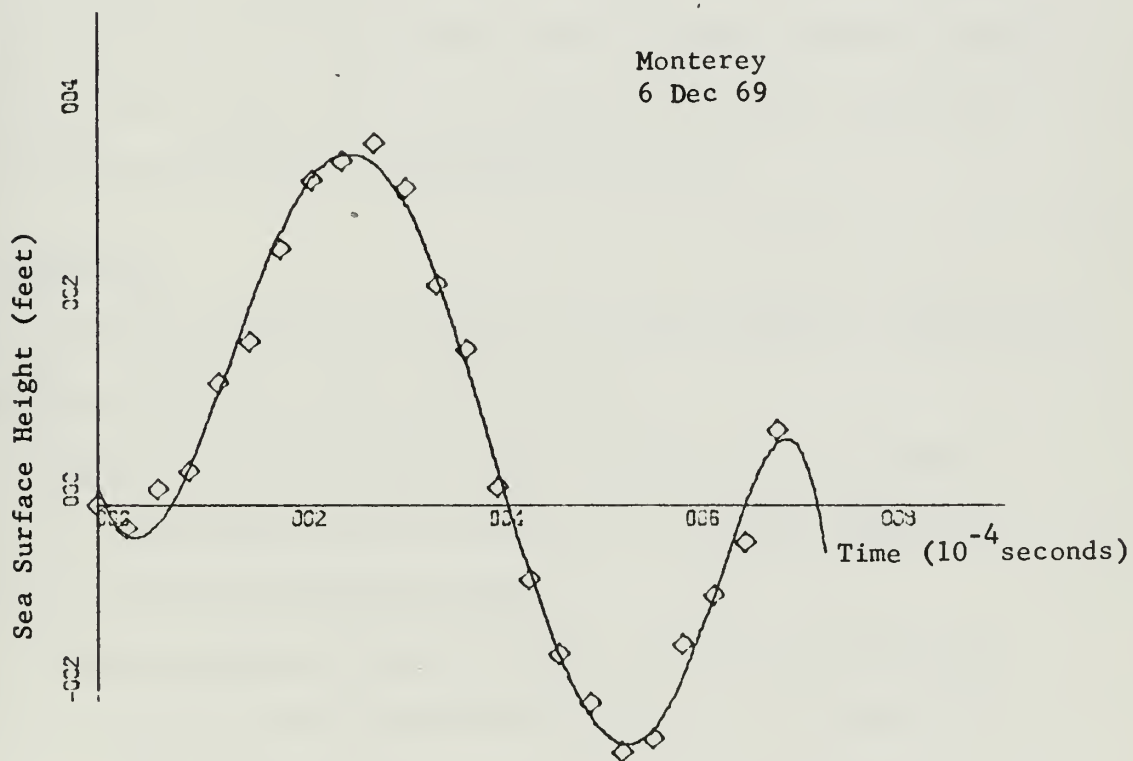
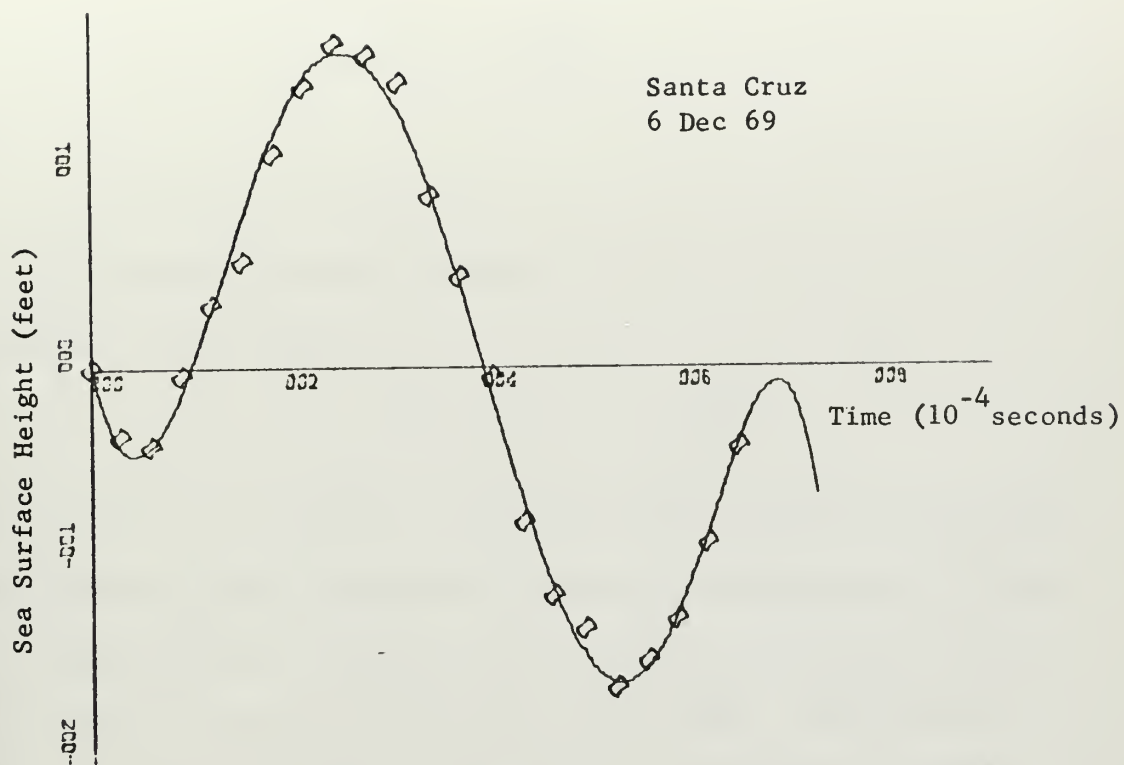


Figure 3

Least Squares Curve for Santa Cruz and Monterey, 6 Dec 69



$$E_j = (A_j^2 + B_j^2) \left( \frac{T}{4} \right)$$

where

$T$  = record length (seconds)

$E_j$  = spectral estimate ( $m^2$ -sec)

The associated frequencies are

$$f_1 = 0, f_2 = \frac{1}{T}, f_3 = \frac{2}{T} \dots, f_N = \frac{N-1}{T}$$

The raw spectral estimates were then smoothed over five frequencies, giving each new estimate 10-degrees of freedom and increased confidence in the spectral peaks. Smoothing over five spectral estimates removes spurious peaks, but does not smear the actual peaks. The 90% confidence limits for 10 degrees of freedom are interpreted as meaning that one can state with 90% accuracy that the actual spectral estimate is greater than  $.49E_j$  but less than  $1.60E_j$ , as the power spectrum is distributed according to a CHI-SQUARE distribution.

## B. SANTA CRUZ

The analysis procedure of the Santa Cruz data was more complex than the Monterey data due to two considerations:

- (1) The Santa Cruz tidal trace is recorded on curvilinear coordinates, but is accepted as rectilinear coordinates by the CALMA 480 digitizer.
- (2) The sampling interval of Santa Cruz is 6 seconds which must be interpolated and matched point for point in time with the Monterey data, in order to compute the cross spectra.



The unadjusted amplitudes and time increments were obtained from use of the CAIMA 480 digitizer, and computer program CONVERT. The analog record was recorded on Bristol Company chart 6112, shown in Figure 4, which has curvilinear coordinates (t,Rθ). The digitizer then recorded incremental pairs, in the order X and Y (of the cartesian coordinates of the tidal curves), every .01 inch of stylus travel. It was therefore necessary to form the cartesian coordinates by summation within the computer. These cartesian coordinates were then converted to the approximately correct T, Rθ coordinates by employing geometric relationships.

The computation of the correct value of  $t_i$  (time axis) is critical to the conversion. On the time scale, 0.1 inch of record was equivalent to a sample interval of 6-seconds. The summation of X,  $X_i$ , representing the horizontal travel was converted to the t, Rθ coordinates by

$$t_i = X_i + R - \sqrt{R^2 - Y_i^2}$$

where,

$t_i$  = time in curvilinear coordinates

R = radius of curvature for arcs of constant  $t_i$

$Y_i$  = the ordinate on the cartesian scale

$X_i$  = time scale in equally spaced increments of t on the cartesian scale.

These geometric relations are shown in Figure 5 where

$$\theta_i = \text{TAN}^{-1} Y_i / \sqrt{R^2 - Y_i^2}$$

The adjusted heights are then computed as:

$$Y_i = R\theta$$

where  $Y_i$  is now the adjusted sea-surface elevation value. Since the





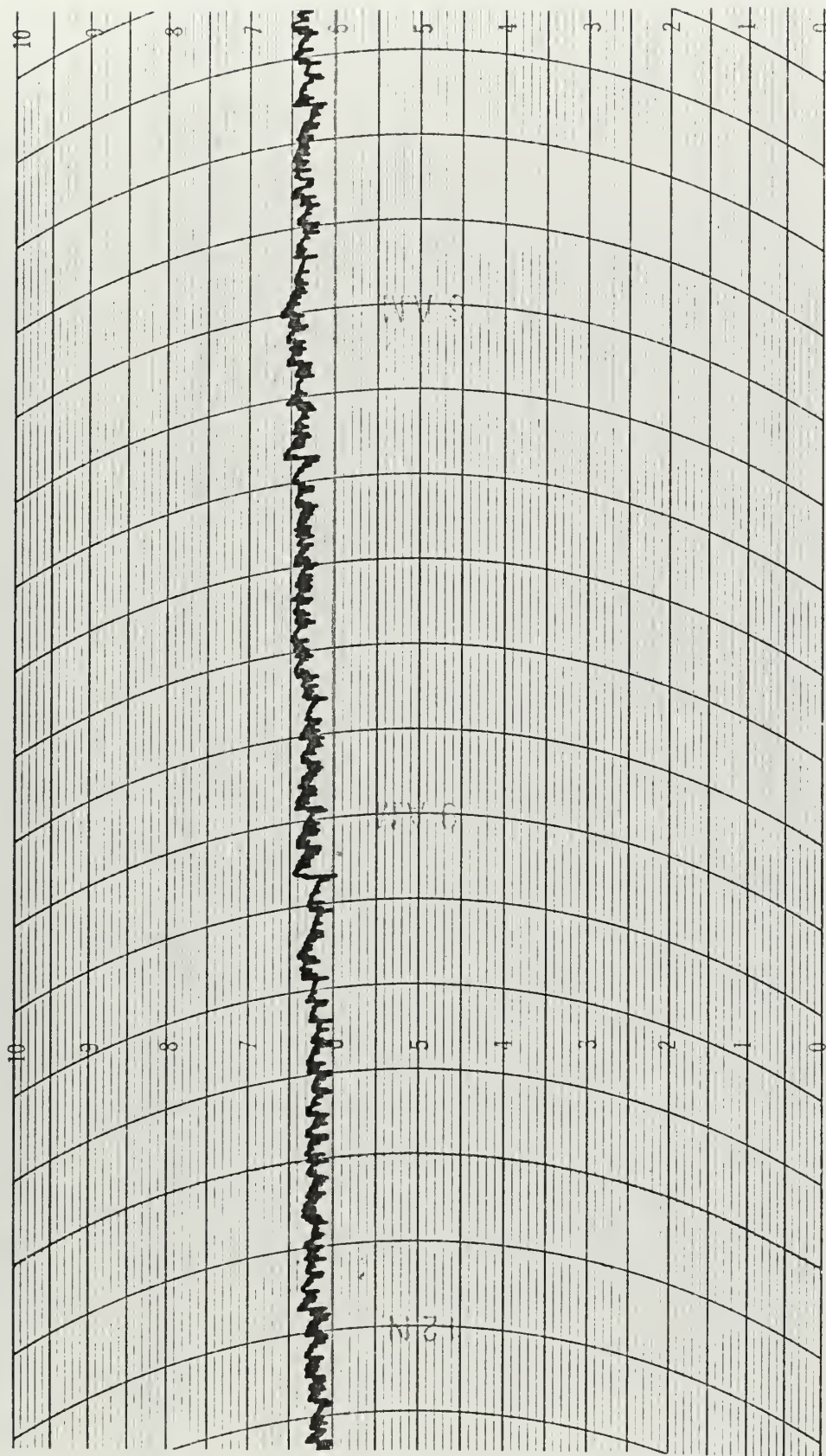


Figure 4  
Bristol Company Chart 6112



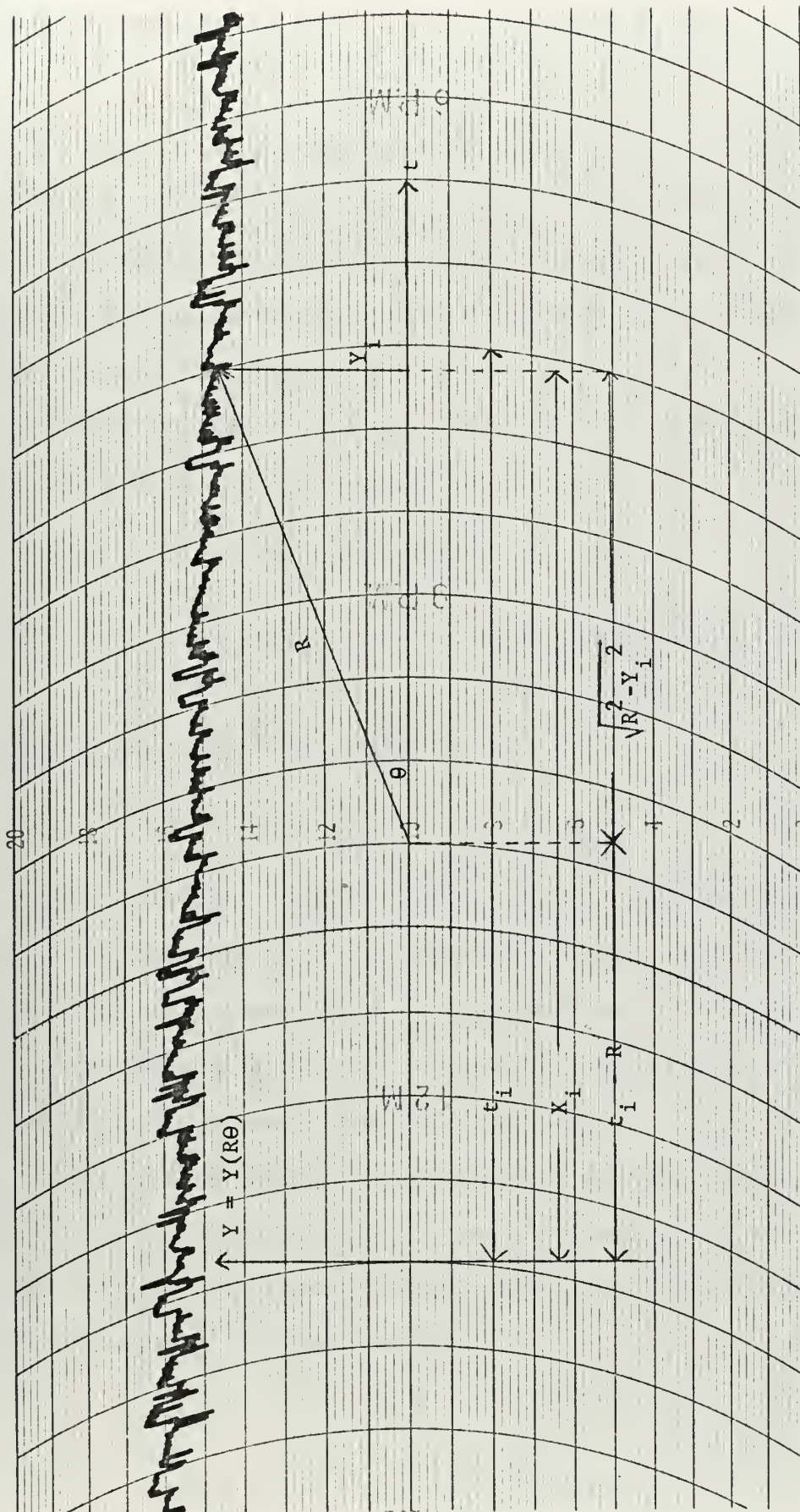


Figure 5

Geometric Relations for Conversion from Curvilinear Coordinates  $(R, \theta)$  to Rectilinear Coordinates  $(Y, t)$



space increment is .01 inch, the adjusted time,  $t_i$  is

$$t_i = 600 X_i$$

The adjusted sea surface elevation (with reference to an arbitrary level) and adjusted times, were not in general equally spaced. Therefore, an interpolation scheme was used to obtain data points at a sampling interval of 34.95 seconds in order to match the Monterey data, point for point (Figure 6).

A linear interpolation was performed between time increments greater than 34.95 seconds. The slope of the line is

$$\frac{\Delta Y}{\Delta t} = \frac{(Y_i - Y_{i-1})}{(t_i - t_{i-1})}$$

The horizontal spatial scale for the interpolated values is,

$$X_i = (i-1)\Delta \quad \text{where } \Delta = 34.95 \text{ sec.}$$

The resultant interpolated amplitude is,

$$A_i = \frac{\Delta y}{\Delta t} (X_i - t_{i-1}) + Y_{i-1}$$

Effectively, the computer checks each successive time value until the desired interval of 34.95 seconds is exceeded, then interpolates between the value greater than 34.95 seconds and the prior value which has an increment less than 34.95 seconds. Santa Cruz data were then aligned with the Monterey data point for point.

The first 2048 points of the Santa Cruz data were handled in the same manner as the Monterey data. The tidal influences were removed by the least-squares scheme, and the adjusted data were analyzed in the same manner as the Monterey data.





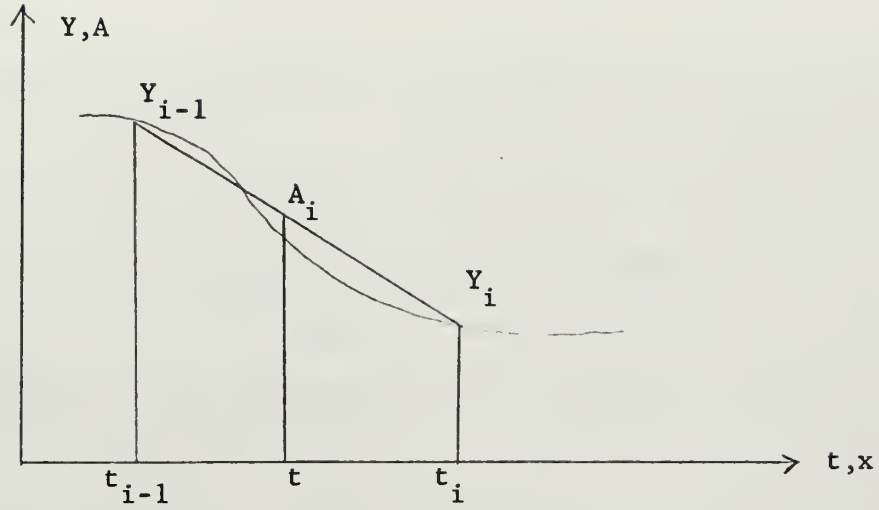


Figure 6

Linear Interpolation

### C. CROSS SPECTRA

Given two random, stationary time-series,  $f_1(t)$  and  $f_2(t)$  the cross-spectra is defined as:

$$\Phi_{12}(\sigma) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \varphi_{12}(\tau) e^{-i\sigma\tau} d\tau \quad (1)$$

where,

$\tau$  = time lag

$\sigma$  = angular frequency

$\varphi_{12}$  = the cross-covariance between  $f_1(t)$  and  $f_2(t)$  given by

$$\varphi_{12}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} f_1(t) f_2(t+\tau) dt \quad (2)$$





where

$\tau$  = time lag

$T$  = record length

$t$  = time

If the time lag  $\tau$  is replaced by  $-\tau$  and substitution of the dummy variable  $\delta = t - \tau$  we have,

$$\varphi_{12}(-\tau) = \lim_{T \rightarrow \infty} \int_{-T/2}^{T/2} f_1(\delta + \tau) f_2(\delta) d\delta$$

then, the cross-covariance is an even function such that

$$\varphi_{12}(-\tau) = \varphi_{21}(\tau) \quad (3)$$

The cross-spectrum (1) can be written (using Euler's formula)

$$e^{-i\sigma\tau} = \cos \sigma\tau - i \sin \sigma\tau$$

as

$$\Phi_{12}(\sigma) = \frac{1}{2\pi} \left\{ \int_{-\infty}^{\infty} \varphi_{12}(\tau) \cos \sigma\tau d\tau - i \int_{-\infty}^{\infty} \varphi_{12}(\tau) \sin \sigma\tau d\tau \right\}$$

since  $\varphi_{12}(\tau)$  is not, in general, an even function, therefore, the cross spectrum is a complex function of  $\sigma$ .

The cross spectrum combines both the cosine and sine transforms of  $\varphi_{12}(\tau)$ . From this, if the cross-spectrum can be resolved into odd and even complements, it can then be written as,

$$\Phi_{12}(\sigma) = \frac{1}{2\pi} \left\{ \int_{-\infty}^{\infty} \varphi_{\text{even}}(\tau) \cos \sigma\tau d\tau - i \int_{-\infty}^{\infty} \varphi_{\text{odd}}(\tau) \sin \sigma\tau d\tau \right\} \quad (4)$$

since the cosine (sine) transform of an odd (even) function is zero.

For convenience we define two functions  $A(\tau)$  and  $B(\tau)$  where



$$A_{12}(\tau) = \frac{\varphi_{12}(\tau) - \varphi_{21}(\tau)}{2}$$

$$B_{12}(\tau) = \frac{\varphi_{12}(\tau) + \varphi_{21}(\tau)}{2} .$$
(5)

If  $\tau$  is replaced by  $-\tau$  in (5), and using (3) gives  $A_{12}(-\tau) = -A_{12}(\tau)$  and  $B_{12}(-\tau) = B_{12}(\tau)$  where A is an odd function and B even. Since  $\varphi_{12}(\tau)$  is resolved as

$$\varphi_{12}(\tau) = A_{12}(\tau) + B_{12}(\tau)$$

we see A and B are the odd and even parts respectively of  $\varphi_{12}$ .

Using this equation, (4) can be written as

$$\Phi_{12}(\sigma) = \frac{1}{2\pi} \left\{ \int_{-\infty}^{\infty} B_{12}(\tau) \cos \sigma\tau d\tau - i \int_{-\infty}^{\infty} A_{12}(\tau) \sin \sigma\tau d\tau \right\} .$$
(6)

The co-spectrum is defined as the real part of (6)

$$C_{12}(\sigma) = \frac{1}{2\pi} \int_{-\infty}^{\infty} B_{12}(\tau) \cos \sigma\tau d\tau$$

while the negative of this imaginary part defines the quadrature-spectrum

$$Q_{12}(\sigma) = - \frac{1}{2\pi} \int_{-\infty}^{\infty} A_{12}(\tau) \sin \sigma\tau d\tau .$$

The cross spectrum is then written in complex form

$$\Phi_{12}(\sigma) = C_{12}(\sigma) - iQ_{12}(\sigma)$$
(7)

If a time series is correlated with itself the resulting co-variance function is even. The cosine transform of the auto-covariance function is the power spectrum of the record. The co-spectrum is the cosine transform of the even part of the cross-covariance function, and gives the amount of power due to in-phase ( $0^\circ$ ) or out of phase  $\pm$



180° components between the two records. The quadrature spectra is similar, however, the components are out of phase by a constant amount.

The spectral phase difference  $\epsilon_{12}(\sigma)$  is defined by

$$\epsilon_{12}(\sigma) = \text{TAN}^{-1} \frac{Q_{12}(\sigma)}{C_{12}(\sigma)}$$

and is the argument of the complex conjugate of the spectrum.

In this study the Fast Fourier Transform method was used to determine the power spectra vice the auto-correlation method. This resulted in a slightly different scheme of calculating the cross-spectra, as described below.

From subroutine RHARM the Fourier coefficients

$$\frac{A_0}{2}, B_0 = 0, \frac{A_1}{2}, \frac{B_1}{2}, \dots, \frac{A_N}{2}, B_N = 0$$

were determined for both the Monterey and Santa Cruz data.

From the Fourier coefficients we form the modules for each station,

$$F_M(j) = \frac{a_j}{2} + \frac{ib_j}{2} = |F_M| e^{i\epsilon_1} \quad (\text{Monterey})$$

and

$$F_S(j) = \frac{A_j}{2} + \frac{iB_j}{2} = |F_S| e^{i\epsilon_2} \quad (\text{Santa Cruz}),$$

or

$$\overline{F_S(j)} = F_S(j) e^{-i\epsilon_2} = \frac{A_j}{2} - \frac{iB_j}{2} \quad (\text{complex conjugate})$$

where  $j = 0, 1, 2, \dots, N$ .

Then the cross-correlation function,

$$\varphi_{12}(\tau) = \sum_{j=1}^{2N-1} \overline{F_S(j)} F_M(j) e^{ij\sigma_1\tau}$$

where  $\sigma_1 = 2\pi/N \, dt$



and the cross spectra

$$\Phi_{12}(j) = F_M(j) \overline{F_S(j)} = \frac{1}{T} \int_{-T/2}^{T/2} \varphi_{12}(\tau) e^{-ij\sigma_1\tau} d\tau$$

form a Fourier integral pair.

The cross-spectra  $\Phi_{12}(j)$  can be expanded as

$$\begin{aligned} \Phi_{12}(j) &= F_M(j) \overline{F_S(j)} = |F_M(j)| e^{i\epsilon_1} |F_S(j)| e^{-i\epsilon_2} \\ &= |F_M(j)| |F_S(j)| e^{i(\epsilon_1 - \epsilon_2)} \end{aligned}$$

where

$$\epsilon_1 = \text{TAN}^{-1} \left( \frac{b_j}{a_j} \right) \quad (\text{Monterey})$$

$$\epsilon_2 = \text{TAN}^{-1} \left( \frac{-B_j}{A_j} \right). \quad (\text{Santa Cruz})$$

By use of Euler's formula,

$$|A| e^{i\theta} = |A| (\cos \theta + i \sin \theta)$$

we have, expanding  $\Phi_{12}(j)$ ,

$$\begin{aligned} \Phi_{12}(j) &= |F_M(j)| |F_S(j)| \cos(\epsilon_1 - \epsilon_2) \\ &\quad + i |F_M(j)| |F_S(j)| \sin(\epsilon_1 - \epsilon_2), \end{aligned}$$

or,  $\Phi_{12} = [\text{co-spectra (in phase)}] + i [\text{quad-spectra (out of phase)}]$ .

For the co-spectra, if  $\epsilon_1 = \epsilon_2$ ,  $\cos(0) = 1$ , and we have maximum value for in phase. If  $\epsilon_1 = \epsilon_2 + \pi/2$  in the quad spectra,  $\sin \pi/2 = 1$  and we have maximum out of phase. If we arbitrarily let  $\epsilon_2 = 0$  then

$$\tan \epsilon_1 = \frac{\sin \epsilon_1}{\cos \epsilon_1} = \frac{\text{QUAD}}{\text{CO}}$$

and the phase difference,

$$\epsilon = \text{TAN}^{-1} \left( \frac{\text{QUAD}}{\text{CO}} \right).$$





From this, the co-spectra, quad-spectra, cross-spectra and phase difference was computed for each spectral component.



#### IV. ANALYSIS RESULTS

##### A. IDENTIFICATION OF LONG WAVES PRESENT

Spectral analysis of ten tidal records indicated that the long-wave activity in Monterey Bay is characterized by a few specific spectral peaks which dominate the spectra. Their period identification and possible causes are explained below.

##### 51.8 Minute Period

According to Wilson (1965), the fundamental mode of longitudinal oscillation of the north-south extremities of Monterey Bay is 44.2 minutes. If the bay is approximated as an enclosed basin of uniform depth and length, (Figure 7), the periods of oscillation are roughly calculated in the following manner

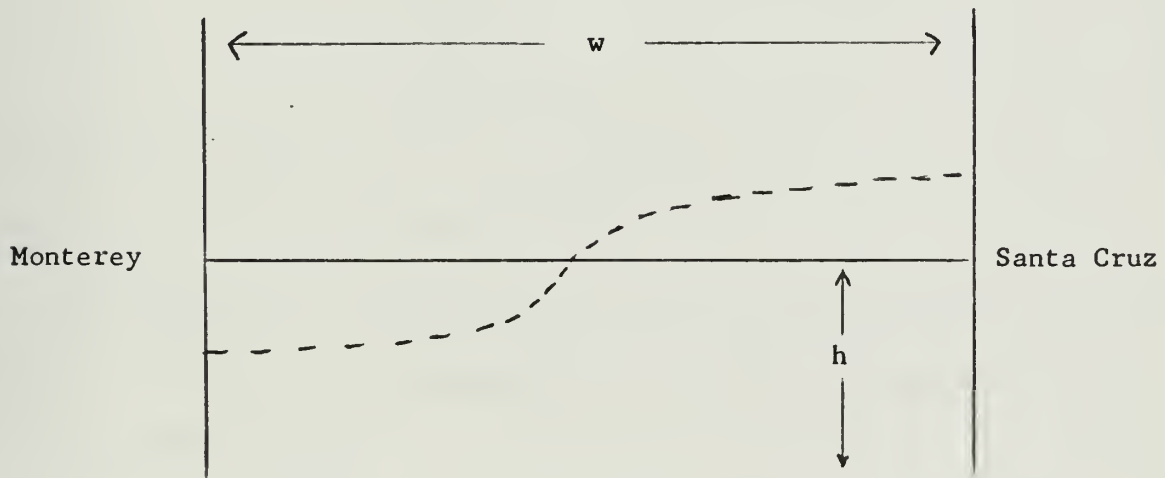


Figure 7

Fundamental Longitudinal Oscillation of Monterey Bay



The shallow-water wave celerity is approximated by,

$$C = \sqrt{gh}$$

where,

C = wave celerity

g = acceleration due to gravity

h = average depth of bay

The wave length of the fundamental period is

$$L = 2w$$

letting  $C = L/T$ , and substituting gives,

$$T_1 = \frac{2w}{\sqrt{gh}}$$

or in general

$$T_n = \frac{1}{n} \frac{2w}{\sqrt{gh}} \quad n = 1, 2, \dots$$

for the harmonics of the fundamental mode of oscillation.

Substituting,

$$w = 20\text{nm}$$

$$h = 240 \text{ feet}$$

$$g = 32.2 \text{ ft/sec}^2$$

gives the fundamental period of oscillation

$$T_1 = 46.5 \text{ minutes}$$

The 51.8 minute long-wave appears throughout the different spectra, and is evaluated as the fundamental period of longitudinal oscillation.

### 36.1 Minute Period

The fundamental mode of transverse oscillation of the east-west extremities of Monterey Bay is computed by Wilson [1965] to be 32.3 minutes. Again, as in all Wilson's calculations, he assumed a nodal



line from Pt. Santa Cruz to Pt. Pinos. From Figure 8 the fundamental transverse oscillation is calculated.

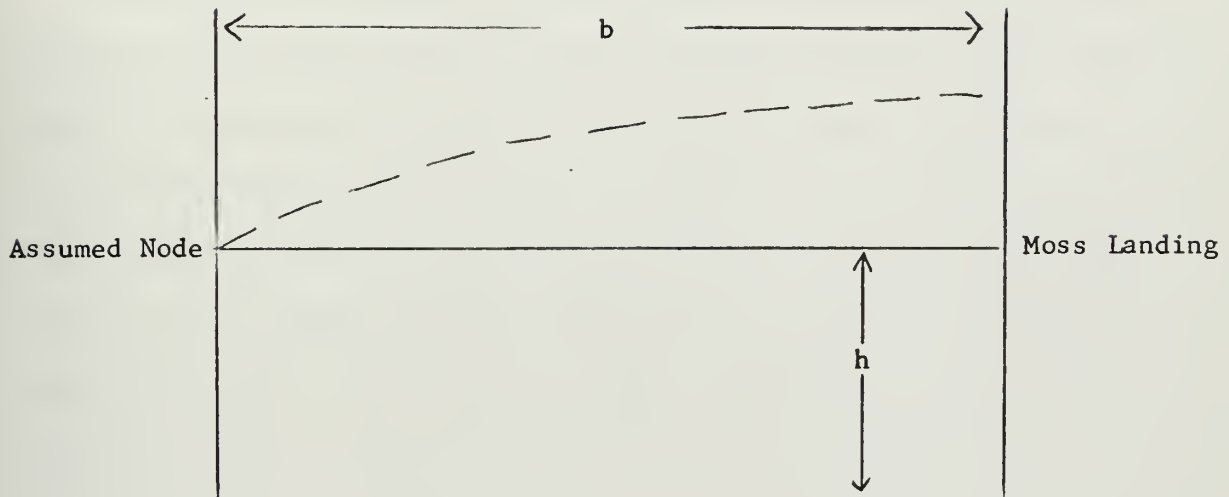


Figure 8

#### Fundamental Transverse Oscillation of Monterey Bay

The fundamental wave length is

$$L = 4b$$

Using the shallow water wave equation

$$C = \sqrt{gh} = \frac{L}{T}$$

and substituting gives,

$$T_n = \frac{1}{n} \frac{4b}{\sqrt{gh}} \quad n = 1, 2, \dots$$

For Monterey Bay,

$$b = 8\text{nm}$$

$$h = 240 \text{ ft}$$

the fundamental period is calculated as





$$T_1 = 36.3 \text{ minutes.}$$

The calculated fundamental transverse mode, which is in agreement with Wilson is present in Monterey four out of the five days analyzed, however, it only appears in Santa Cruz on 3 December 1969.

In addition to the above, one can also consider the possible presence of significant edge-wave activity [Munk, Snodgrass and Carrier, 1956]. The edge-wave is a wave form in shallow water on sloping shelves and beaches. It is called an edge-wave because the waves approach with the crest perpendicular to the beach and travel along the coast. The waves are sinusoidal, and it is assumed the shelf slope is constant. The celerity of the edge-wave as a function of period and shelf slope is given by,

$$C = \frac{gT}{2\pi} \sin \beta$$

where,

C = wave celerity

T = period of edge-wave

$\beta$  = shelf slope

g = acceleration due to gravity.

Substituting  $C = L/T$  we have the fundamental period and harmonics of the edge-wave

$$T_n = \sqrt{\frac{2\pi}{g} \frac{L}{\sin \beta} (2n-1)} \quad n = 1, 2, \dots$$

where L is the wave length equal to twice the longitudinal distance across the bay for the fundamental mode.

Substituting,

$$g = 32.2 \text{ ft/sec}$$



$$\beta = 600 \text{ ft/8nm}$$

$$\frac{L}{2} = 20\text{nm}$$

the fundamental period of oscillation and associated harmonics of edge-waves for Monterey Bay are computed as,

$$T = 35.8, 25.3, 20.2, 17.8, 15.7, \dots \text{ minutes}$$

These modes are also called "trapped modes" from wave-guide theory as the wave energy is trapped into a continental wave guide where in theory the energy remains trapped indefinitely under idealized conditions.

Although the 35.8 minute long-wave is significantly present as indicated above, it is not evaluated as an edge-wave as phase computations do not support any pronounced edge wave activity.

### 27.7 Minute Period

The 27.7 minute period appears to be the most persistent long-wave in the bay. It appears in the spectrum having significant energy density on four out of five days analyzed, and is present at both Santa Cruz and Monterey on 6 December 1969 and 14 January 1970. The 27.7 minute wave is also the dominant long-wave in Robinson's [1965] analyses. Such waves can be interpreted as shelf-waves [Munk, 1962]. The formula for the case of normal incidence on a two-step topography (Figure 9) consisting of the continental shelf of width A and constant slope S', and the continental slope of width B with slope S is

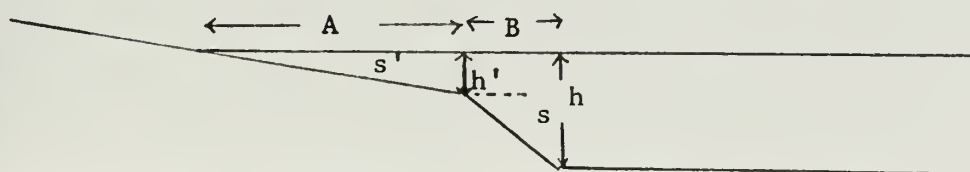


Figure 9

Two-Step Shelf Topography



$$T_0 = \frac{2\pi A}{\beta} \frac{1}{\sqrt{gh}}$$

where,

$T_0$  = fundamental period of the shelf wave

$\beta$  = constant which is dependent upon ratios  $S'/S$  and  $h'/h$

where  $h'/h$  is the ratio of depth of break in continental shelf to depth of sea floor at the bottom of continental slope.

$g$  = acceleration due to gravity.

For the case of Monterey Bay, choosing the 100 fathom curve as  $h'$  and average distance  $A$  equal to 8.0nm and letting  $4\beta/\pi = 1.54$  when  $S' \ll S$  and  $h' \ll h$ , the formula for the shelf-wave fundamental period becomes

$$T_0 = \frac{8A}{(60)1.54} \frac{1}{\sqrt{gh'}}$$

Substituting,

$\bar{A} = 48000$  ft (average distance to 100 fathom curve)

$g = 32.2$  ft/sec<sup>2</sup>

$h' = 600$  ft

we have,

$T_0 = 29.6$  minutes.

Modification of  $A$  and  $h'$  in the shelf-wave formula for different locations around the bay changes the fundamental period only slightly.

The remaining long-waves which are present during significant long-wave activity are those of periods 22.5, 18.9, 16.3, 13.6, 10.1, and 9.3 minutes. These are considered to be harmonics of the basic fundamental period 51.8, 36.1 and 27.7 minutes, and are in general weak in spectral density. The significant spectral peaks found on various dates are summarized in Table VII.



PERIODS (MINUTES)											
DATE	LOCATION	51.8	36.1	27.7	22.5	18.9	16.3	13.6	10.1	9.3	
6 Nov 69	Monterey Santa Cruz	x x	x		x		x x		x x	x x	
3 Dec 69	Monterey Santa Cruz	x	x x	x			x	x x	x x	x	
6 Dec 69	Monterey Santa Cruz	x	x	x x		x	x	x	x x	x	
14 Jan 70	Monterey Santa Cruz	x x		x x							
20 Jan 70	Monterey Santa Cruz	x	x	x							
Robinson [1969]											
23 Jan 70	Monterey Santa Cruz	x	x	x		x x					
20 Apr 70	Monterey Santa Cruz			x x		x		x x	x		

TABLE VII

Summary of Significant Spectral Peaks in Monterey Bay





## B. INTERPRETATION OF INDIVIDUAL SPECTRA

Figures 10, 12, 14, 16, and 18 show the synchronized tidal traces of Santa Cruz and Monterey on dates indicated. The traces are plotted with a sampling interval of one minute which is approximately equal to the Nyquist period of 70 seconds, which is the lowest period which can be analyzed using the 34.495 sampling rate. The tidal traces are reproduced here for the first 16 hours of each period analyzed.

### 6 November 1969

The individual spectra for 6 November 1969 are plotted in Figure 11. The record tends to be a "noisy" record with considerable long-wave activity. The weather conditions in Monterey Bay were calm winds and seas in the early morning, increasing to 15 kts of wind from the southwest about 2000. The surface air temperature was 55°F and skies were overcast for most of the day. The 42.5 minute non-recurring long wave appears to be significant (the term significant peak used in this study is a peak that has significant spectral density when compared with other peaks in the same record vice having a specific value) at both Monterey and Santa Cruz only on this date. Most noticeable in the records are waves of periods 51.8, 42.5, 16.3 and 10.1 minutes which appear at both stations and have significant energy-density.

### 3 December 1969

Figure 13 shows the individual power spectra for Monterey and Santa Cruz from 0130-2130 3 December 1969. Examination of Figure 12 shows that significant wave activity in the bay commenced about 0950. Initially the surface winds were from the north at 2 kts, shifting to south at 15 kts by 1200. Skies were clear and surface air temperature increased from 45°F in the early morning to 58°F in the afternoon. The



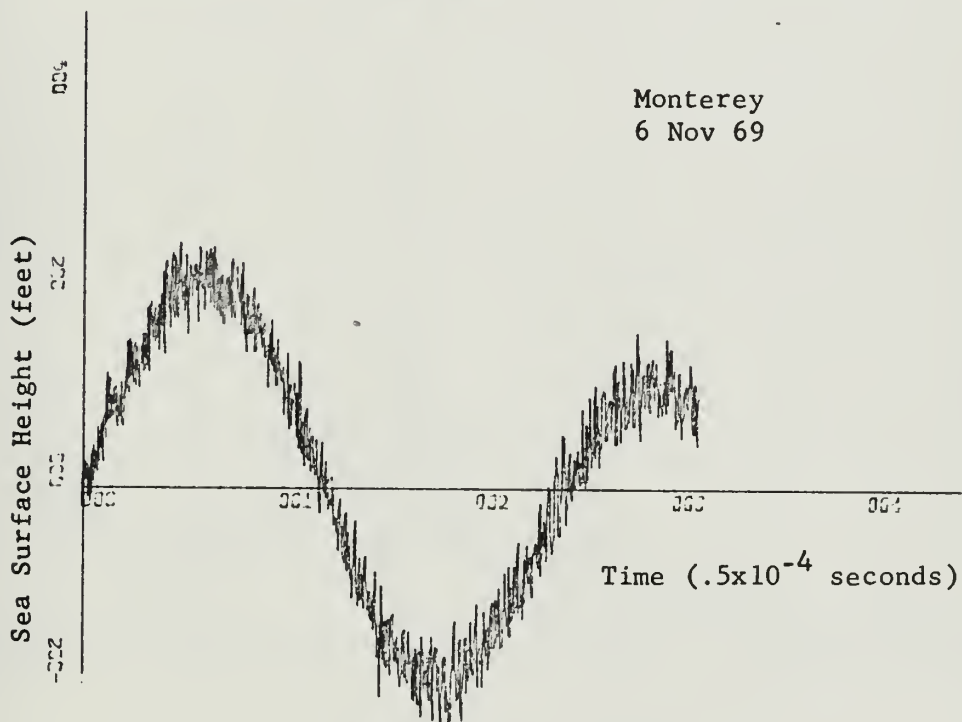
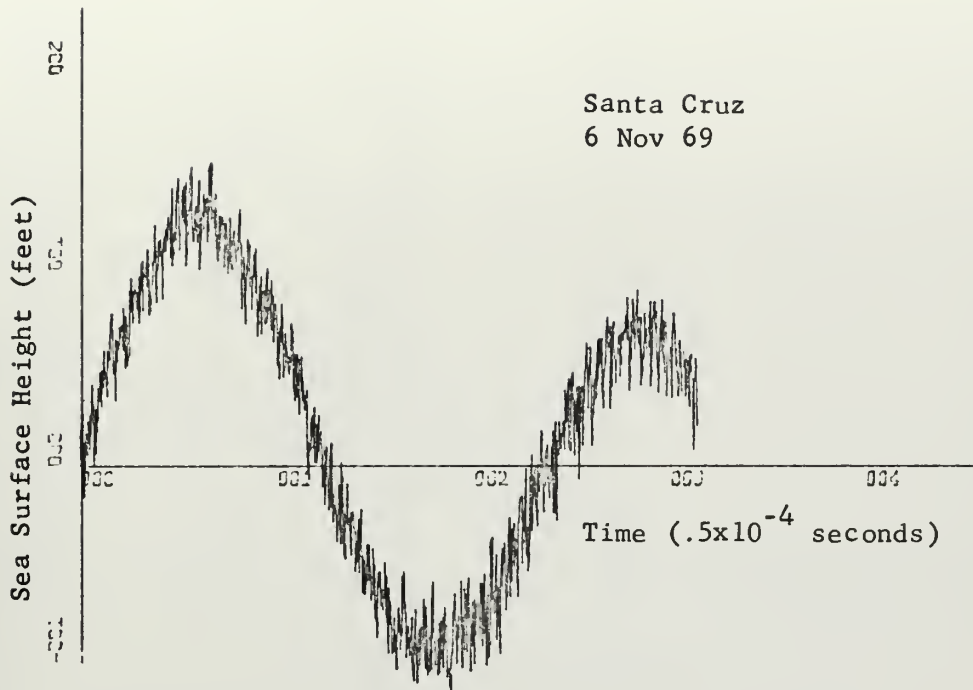
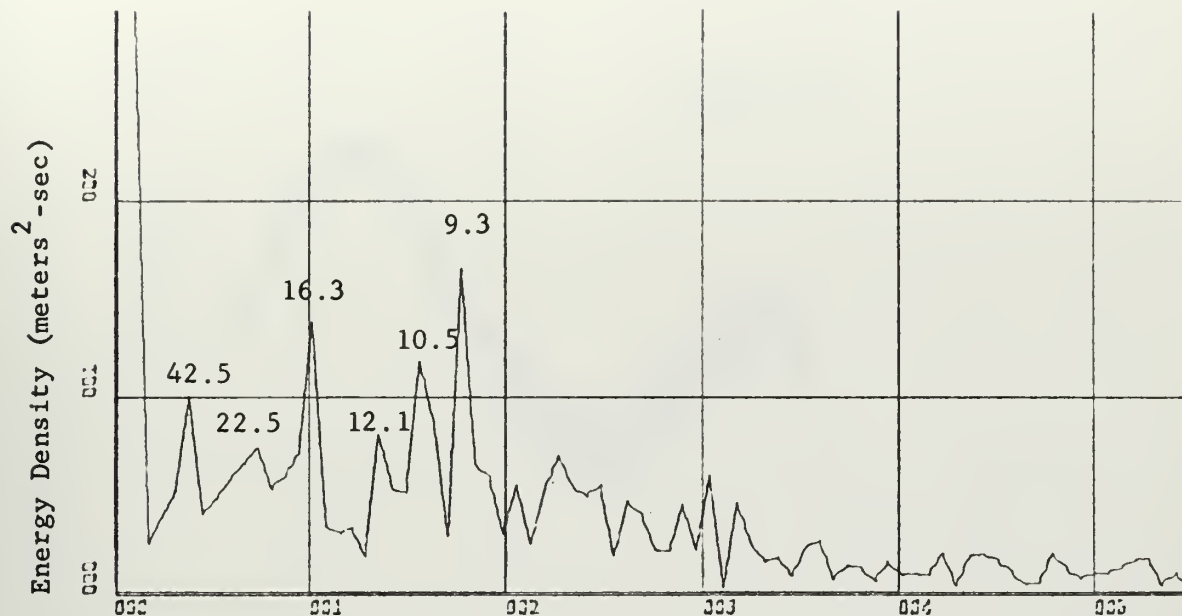


Figure 10

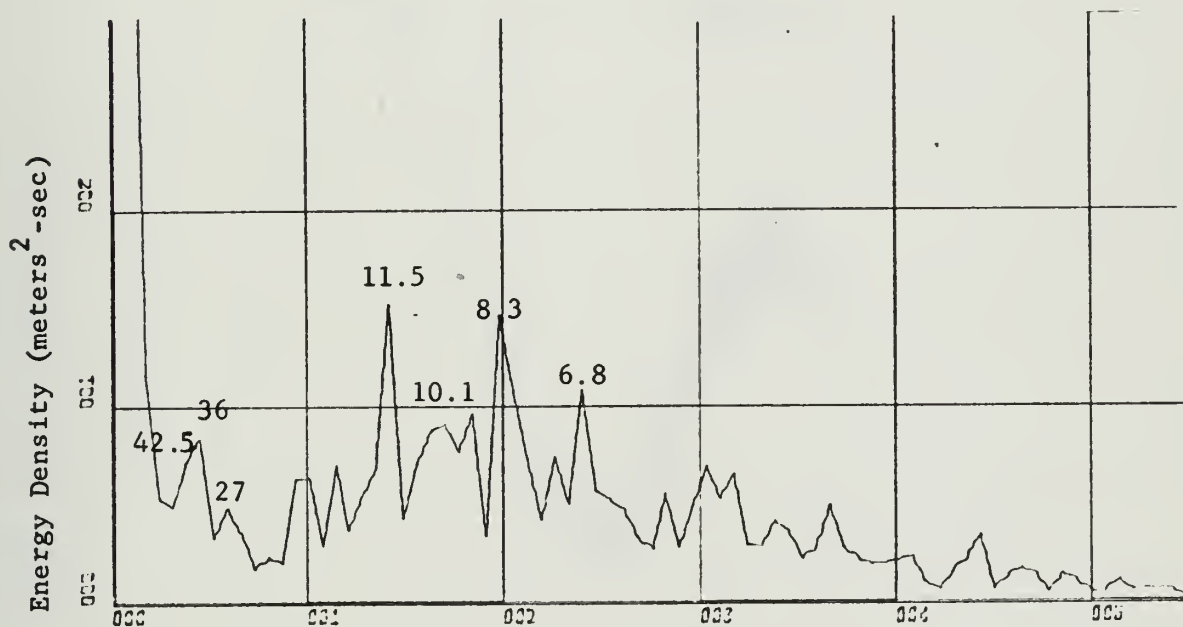
Sea Surface Heights, Santa Cruz/Monterey  
0730 6 November 1969 to 0330 7 November 1969





Frequency (millihertz)

Santa Cruz 0730 - 0330, 6 Nov 69 - 7 Nov 69



Frequency (millihertz)

Monterey 0730 - 0330, 6 Nov 69 - 7 Nov 69

Figure 11

Spectral Wave Analysis, 6 Nov 69



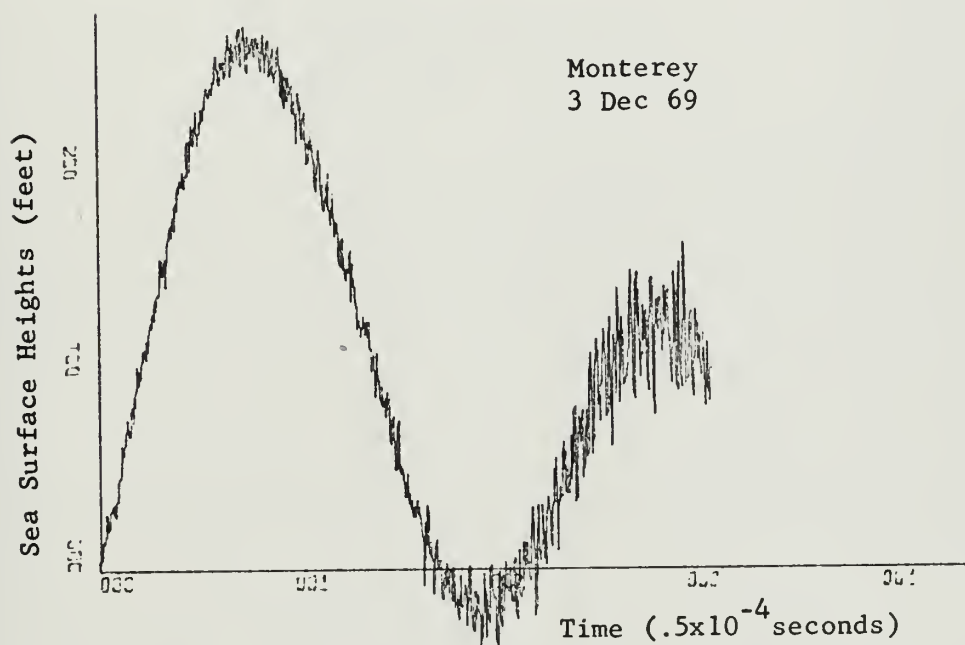
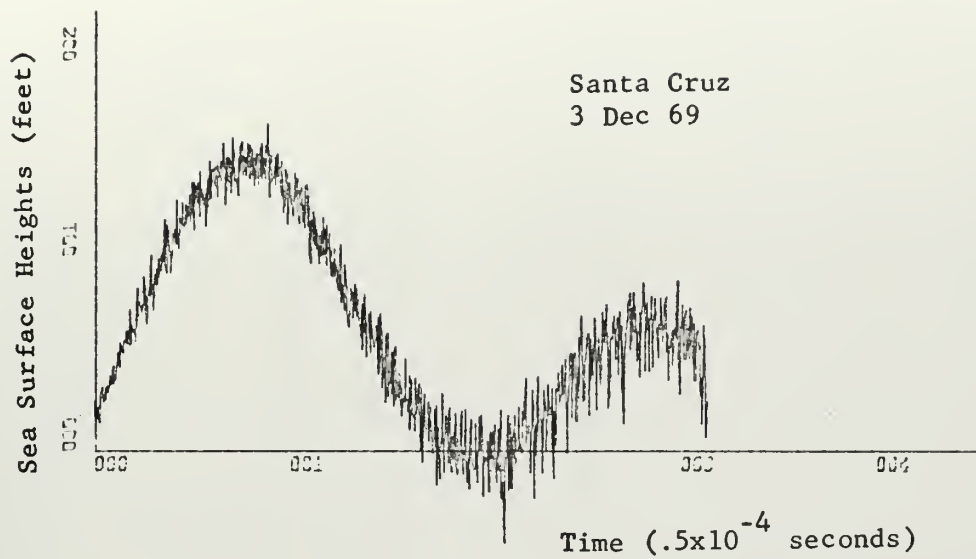


Figure 12

Sea Surface Heights, Santa Cruz/Monterey  
0130 - 2130, 3 Dec 69





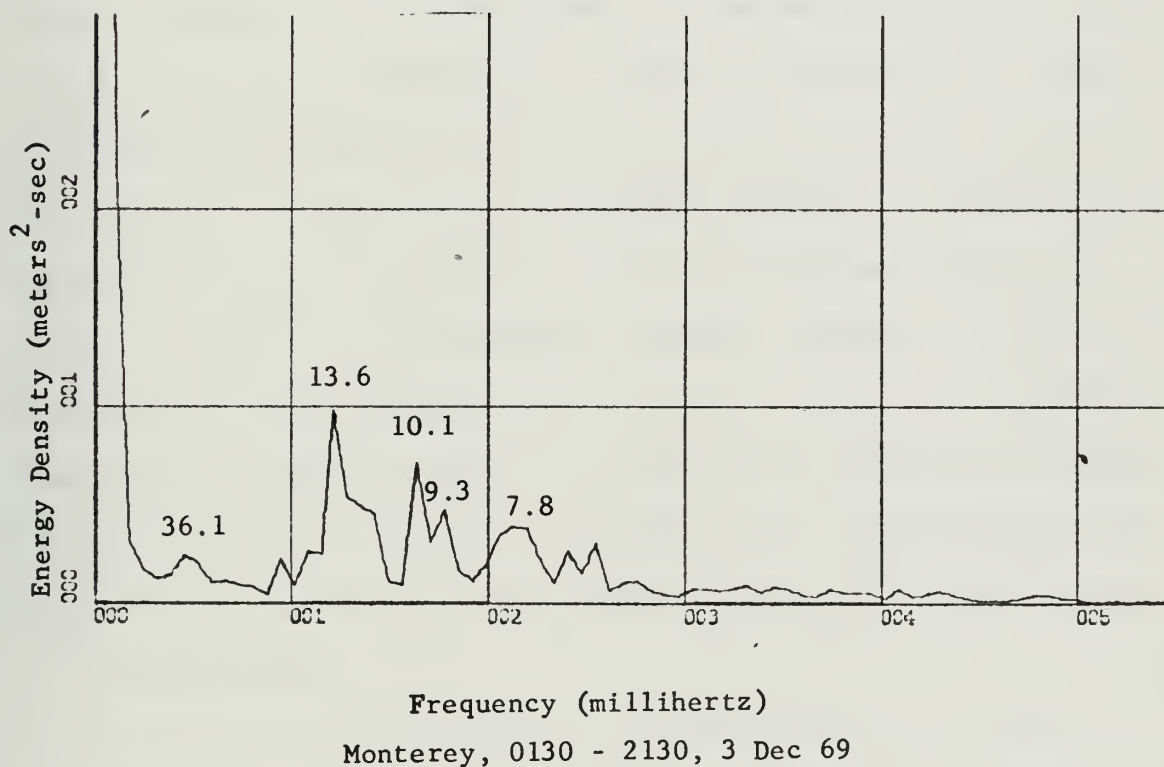
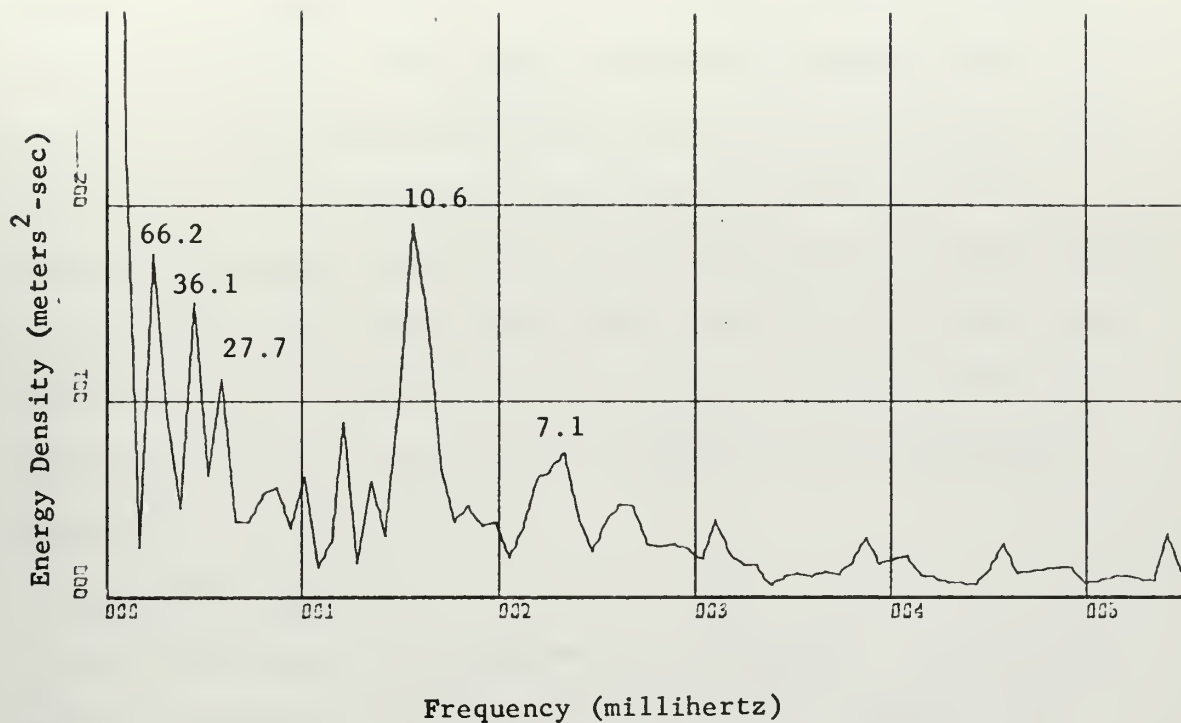


Figure 13

Spectral Wave Analysis, 3 Dec 69



spectra shows intense long-wave activity at Santa Cruz and moderate activity at Monterey. The effect of persistent southerly winds may account for this difference, however, no correlation is drawn. The 36.1, 13.6 and 10.1 minute waves have pronounced energy at both locations, while the 51.8, 27.7 and 16.3 minute waves are significant at Santa Cruz alone. Note, that at Santa Cruz the 13.6 minute wave can very well be a harmonic of the 27.7 minute shelf wave while the 16.3 and 10.1 minute waves could be harmonics of the fundamental transverse mode of 36.2 minutes.

#### 6 December 1969

The tidal traces for 6 December 1969, Figure 14, indicate heavy long-wave activity at Santa Cruz and only light to moderate wave action at Monterey. The winds in Monterey Bay were northwest at 16 kts in the early morning decreasing to 2 kts in the evening. The surface air temperature averaged 55°F and clear skies were observed during the day. Although in contrast to the situation observed on 3 December 1969 no relation of wind to long-wave activity can be drawn during this period. The 27.7 minute shelf-wave is readily apparent in both records with the 13.6 minute harmonic present at Santa Cruz alone. The Santa Cruz picture is quite similar to that experienced earlier on 3 December 1969 at Santa Cruz. Monterey is generally weak in energy density and the shelf-wave appears to be the only active wave of any significance.

#### 14 January 1970

The analyses of wave records on 14 January 1970 are the most distinctive of the study. The sea surface traces, Figure 16, show moderate to heavy wave activity at both stations. During the period analyzed, winds were southerly at 35 kts and heavy rain was falling in the bay.



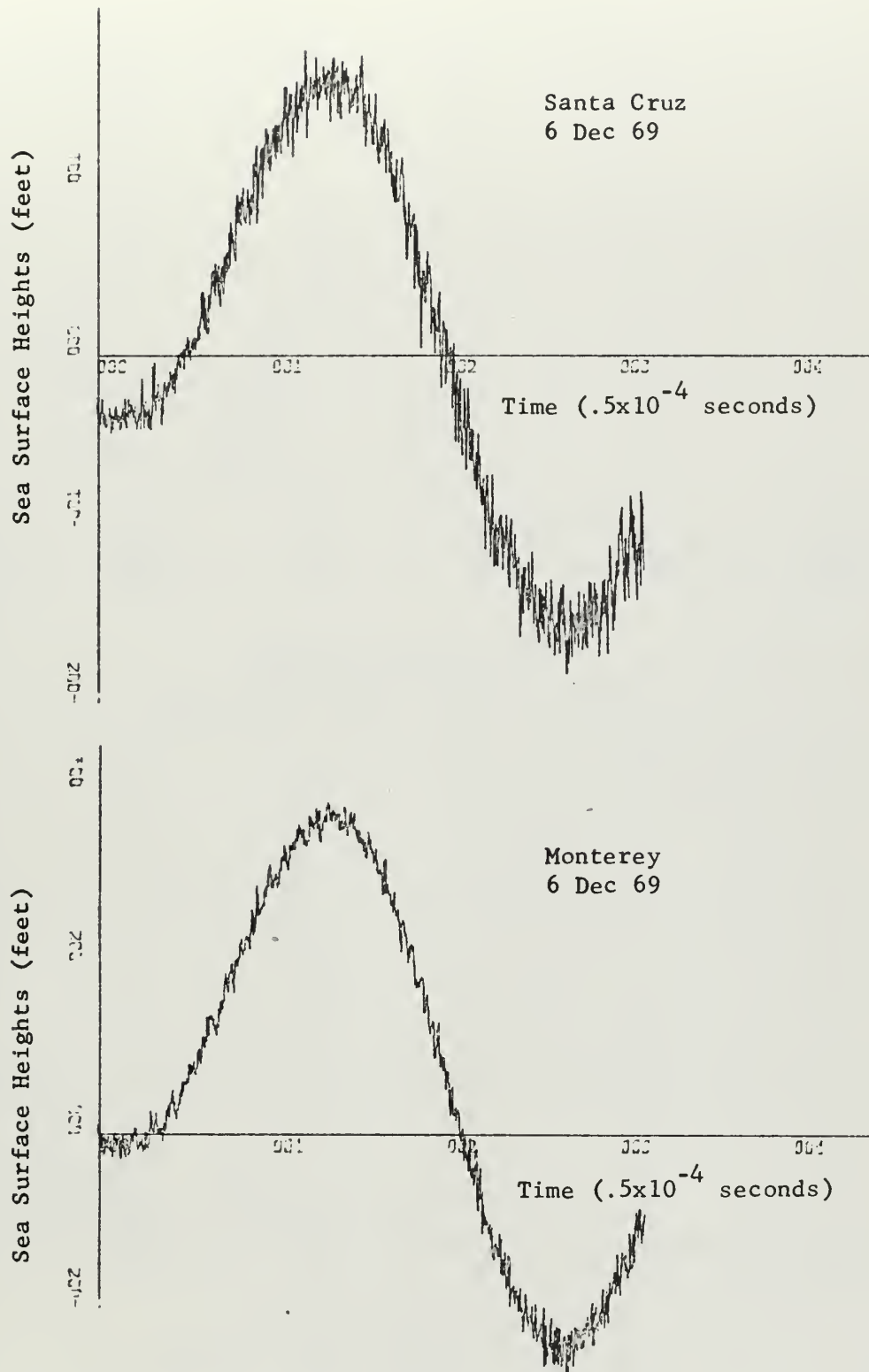
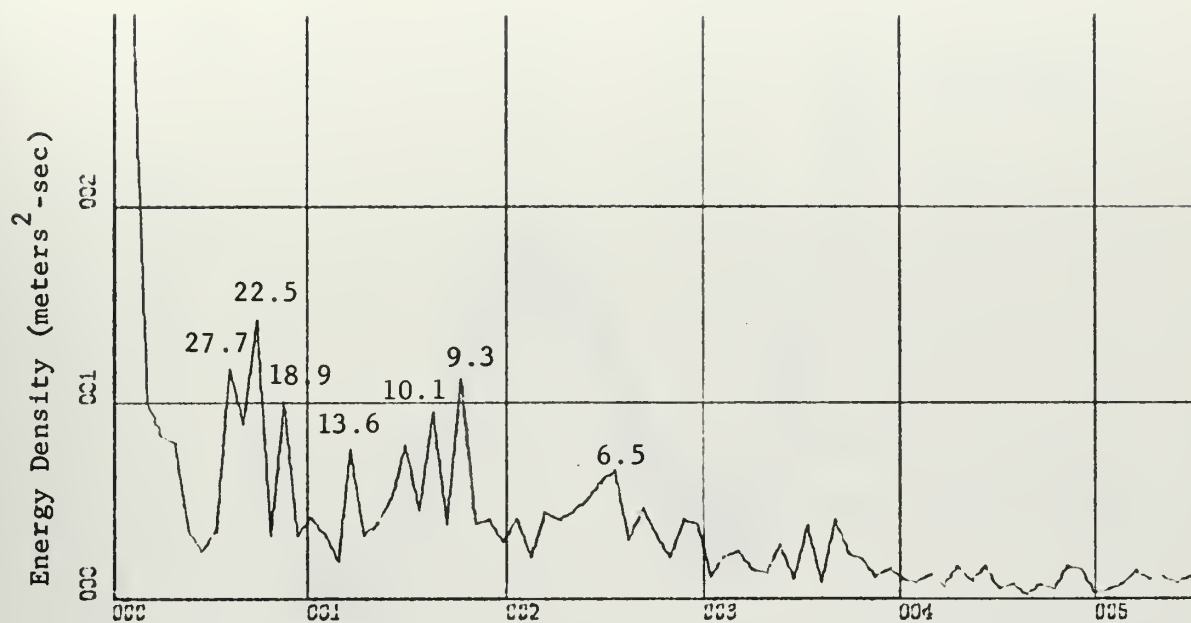


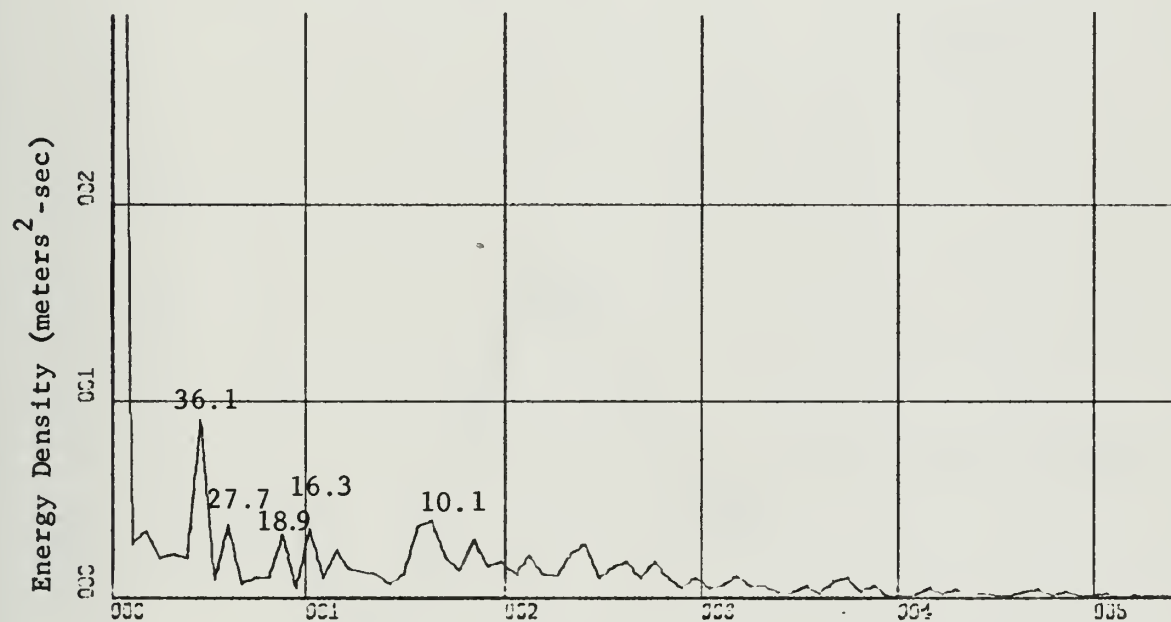
Figure 14

Sea Surface Heights, Santa Cruz/Monterey  
2348 - 1848, 5-6 Dec 69





Frequency (millihertz)  
Santa Cruz, 2348 - 1848, 5-6 Dec 69



Frequency (millihertz)  
Monterey, 2348 - 1848, 5-6 Dec 69

Figure 15

Spectral Wave Analysis, 6 Dec 69





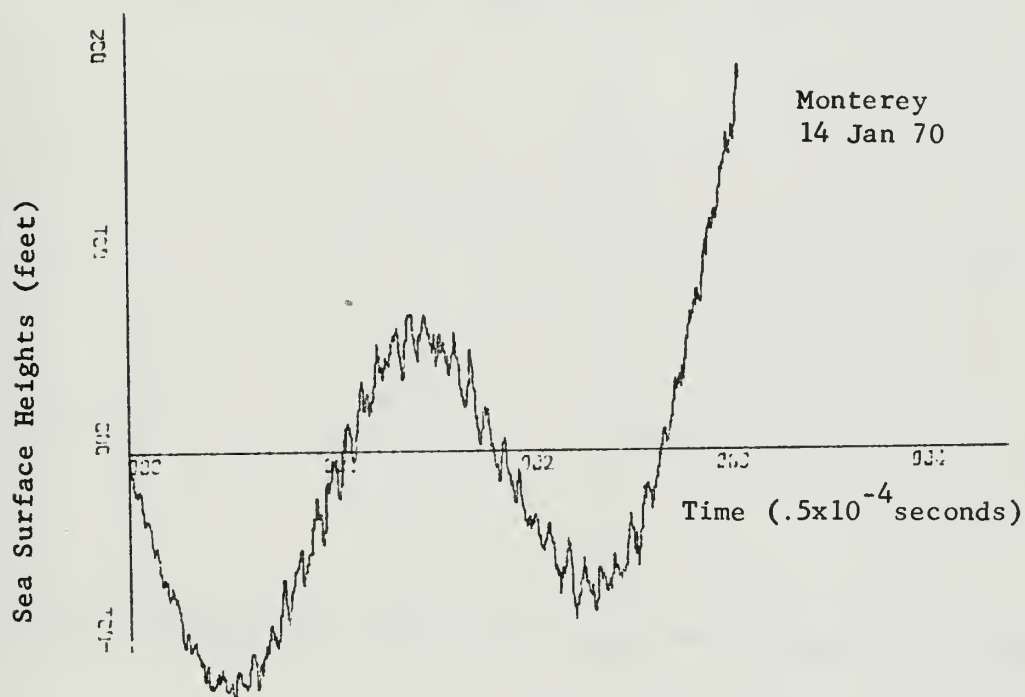
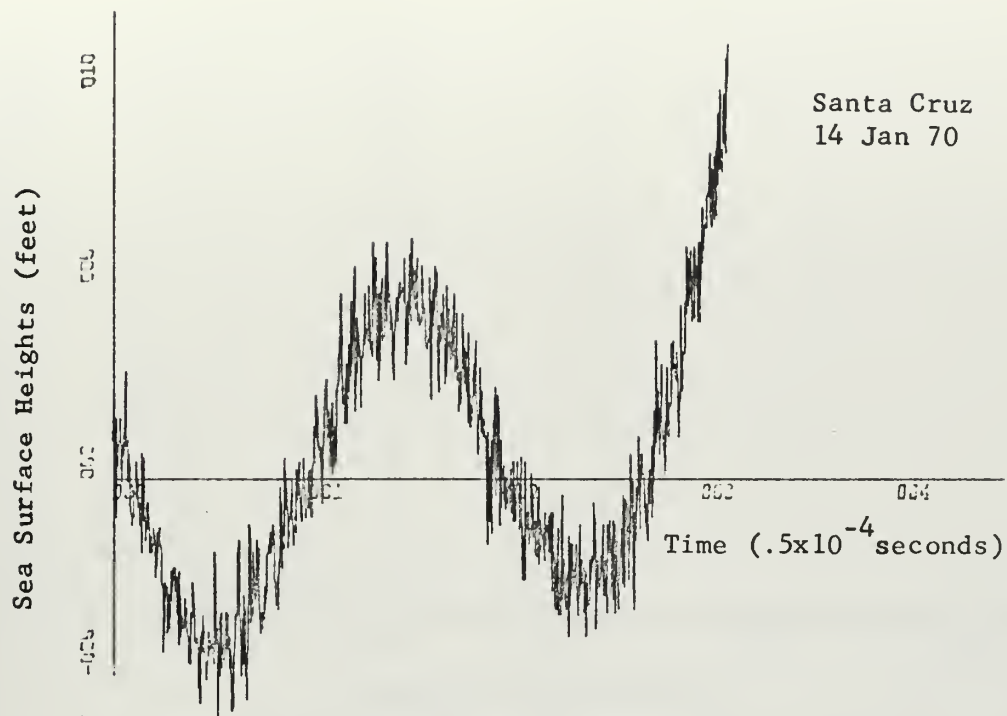


Figure 16

Sea Surface Heights, Santa Cruz/Monterey  
0810 - 0410, 14 Jan 70 - 15 Jan 70



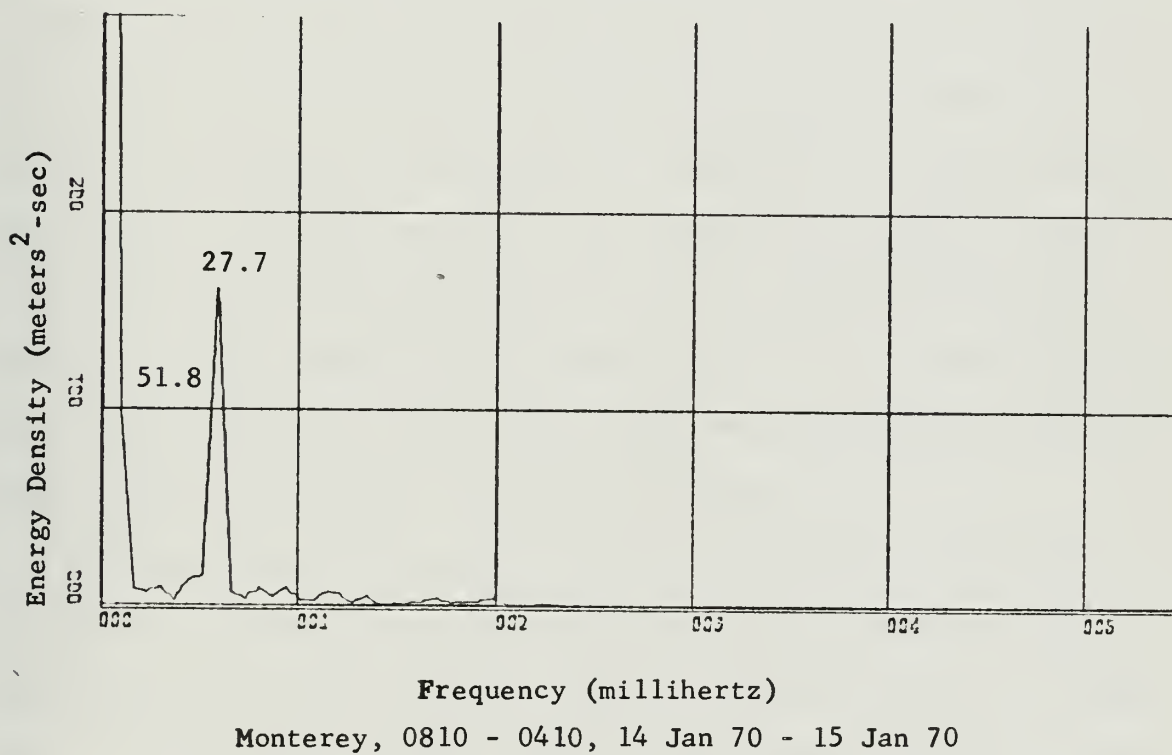
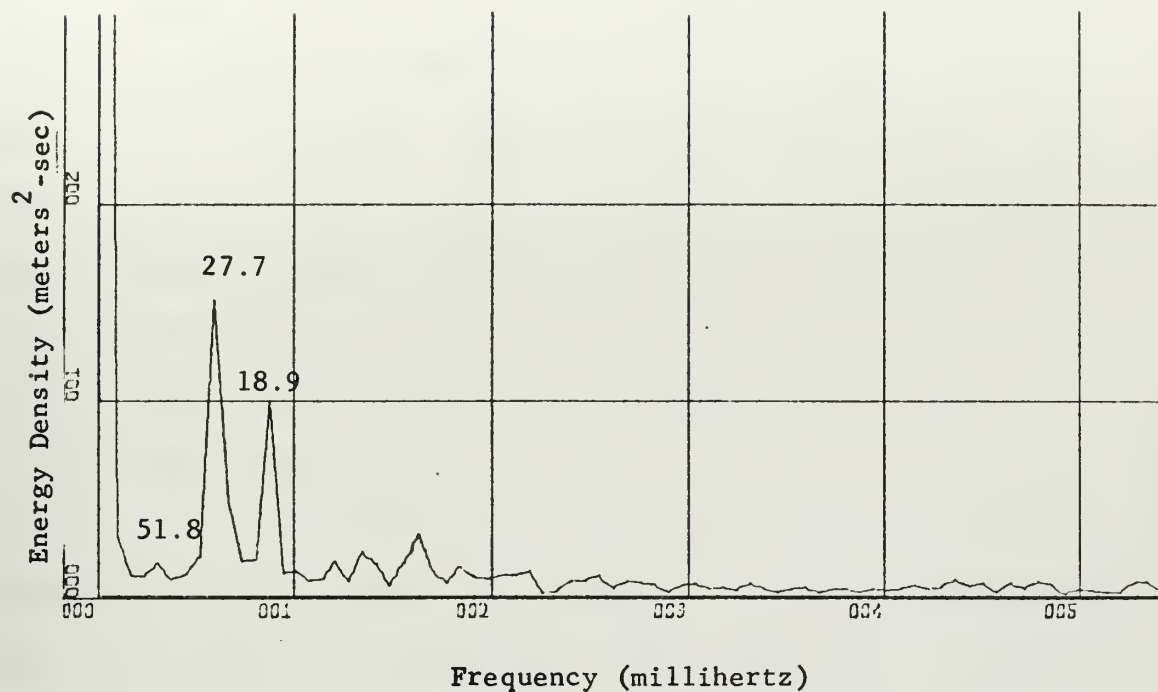


Figure 17

Spectral Wave Analysis, 14 Jan 70



Winds decreased to 2 kts in the late evening, however, surface air temperature remained near 50<sup>0</sup>F during the entire period. The 27.7 minute wave is the primary long-wave present at both locations. In addition the 18.9 minute wave is present at Santa Cruz but extremely weak at Monterey. Again the relationship between weather conditions and long-waves present is not characterized by any definite pattern. The 51.8 minute wave is present at both locations, but is weak.

#### 20 January 1970

Wave activity at Santa Cruz was again more intense than that at Monterey. Winds were light and variable in the morning increasing to southerly at 15 kts with heavy rain in the early evening. The wave activity was similar to that experienced on 3 December 1969 showing moderate southerly winds, heavy wave activity at Santa Cruz and light activity at Monterey. However, with only two examples of this situation, a clear pattern is not observed. The Santa Cruz spectrum shows some long-wave activity present at many discrete frequencies, however, the 51.8 and 13.6 minute periods are the only recurring wave periods present. In this record the 27.7 minute shelf-wave was weak at Santa Cruz. The 36.1 and 27.7 minute waves were observed at Monterey, however, overall wave activity at Monterey was weak.

Looking at the overall picture, including Robinson's [1969] data, it was seen that the 27.7 minute wave evaluated as a shelf-wave, was the dominant long-wave in the bay. This 27.7 minute period is averaged over 5 frequency band widths therefore appears within the range of 26.3 to 29.4 minutes. Various values of average depth of the shelf and distance to the shelf, in general, yield periods which fall within this range. The fundamental transverse period of oscillation was observed



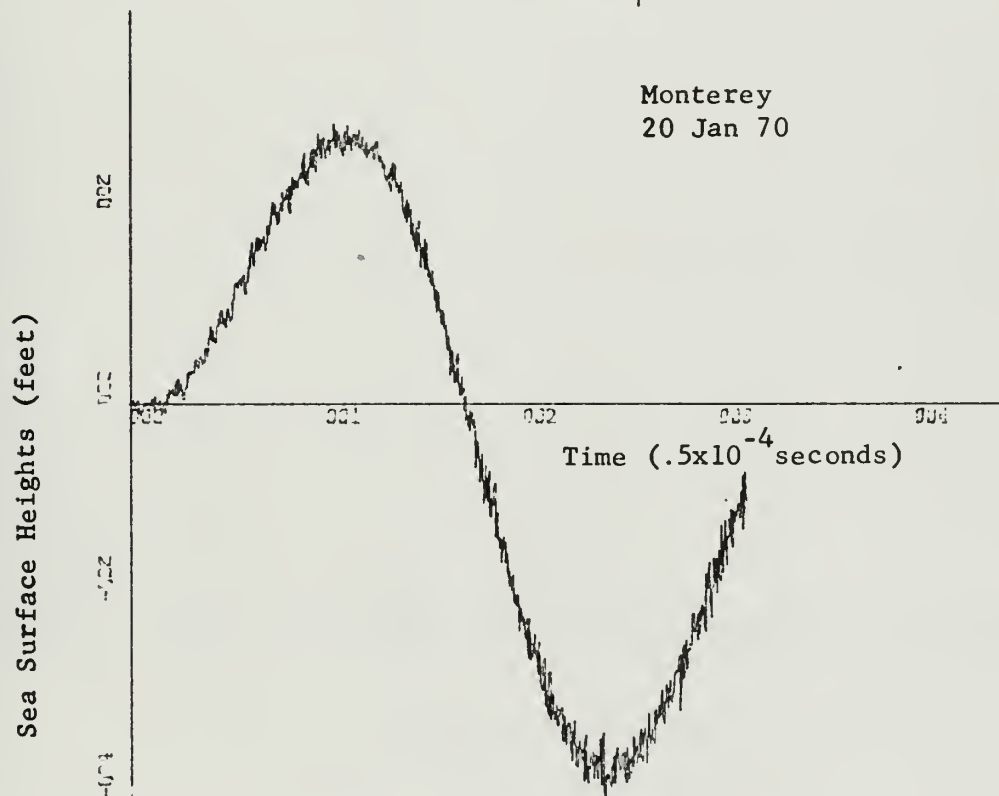
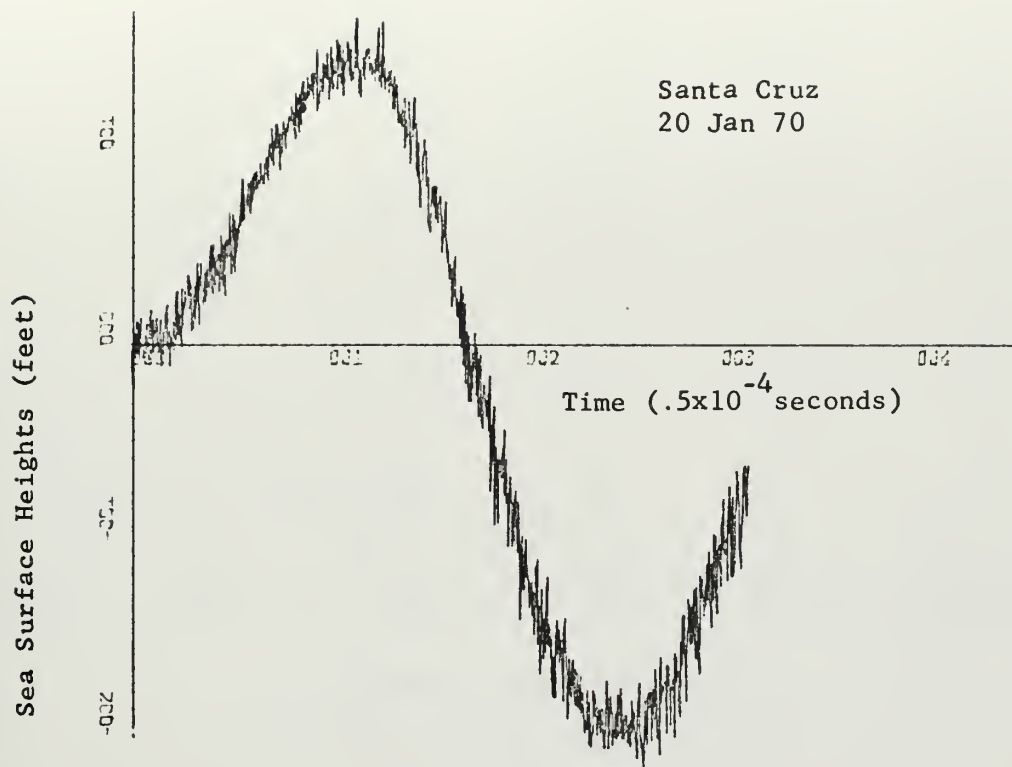


Figure 18

Sea Surface Heights, Santa Cruz/Monterey  
0215 - 2215, 20 Jan 70





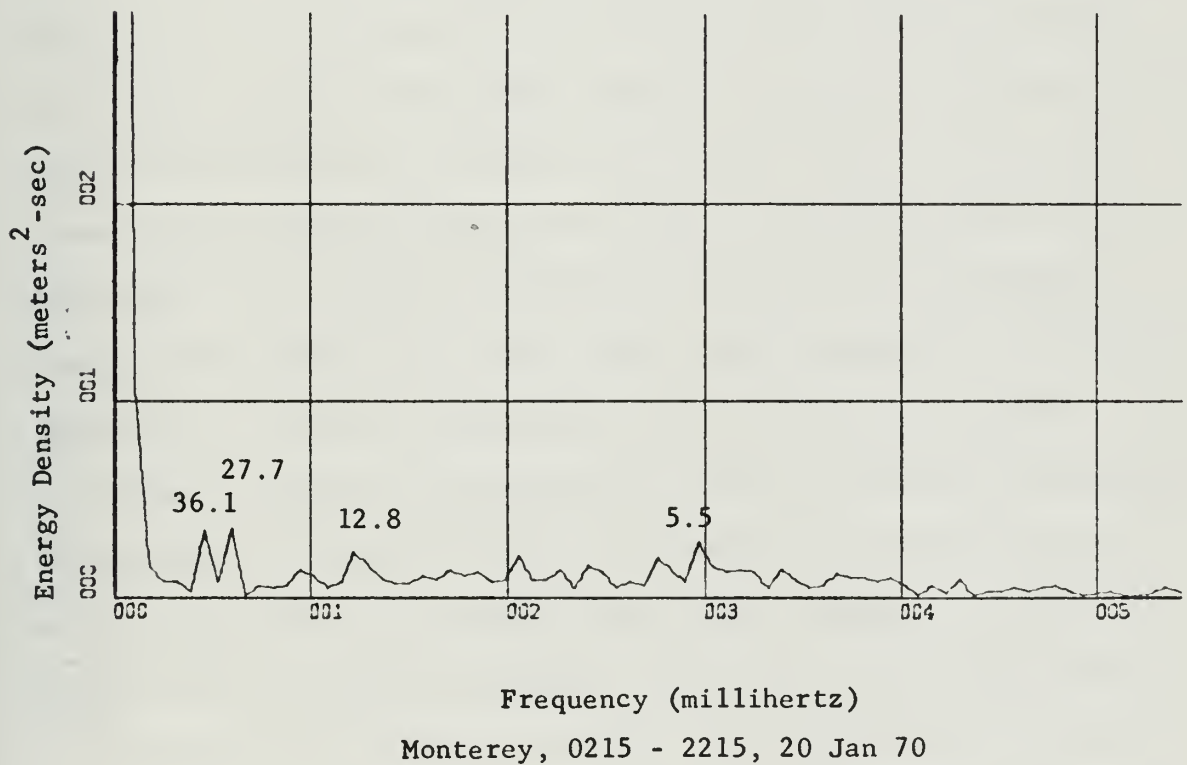
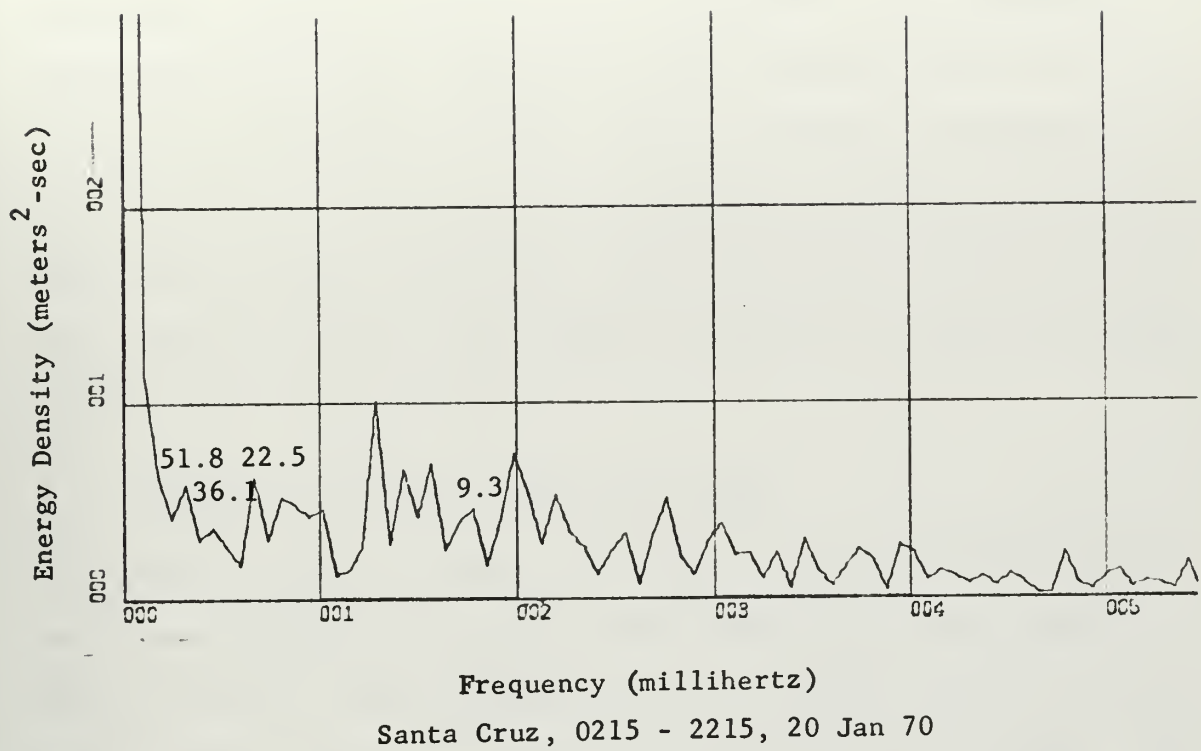


Figure 19

Spectral Wave Analysis, 20 Jan 70



regularly at Monterey, but only once at Santa Cruz. The fundamental longitudinal period was significant at both locations most of the time. No definite relationship was drawn between atmospheric conditions and long-wave activity, however, a strong southerly wind seemed to lead to heavy wave activity in Santa Cruz. Also it was noted that most of the time, long-wave activity when present in the bay, was much more intense at Santa Cruz than at Monterey.

### C. INTERPRETATION OF CROSS SPECTRA AND PHASE

Table VIII gives a summary of selected recurring long-period waves which have significant cross power when compared with other peaks in the same record. Some high energy-density long-waves which only appear once or twice in the entire study are excluded from the analyses. The cross spectra is a measure of the combined energy-density resulting from contributions of each power spectra but gives no information as to the forcing function of the wave or to whether the waves at the two locations are correlated with each other. Two isolated waves of the same period but operating independently, each having considerable energy, would yield a high cross power. Phase relationships determine whether or not the waves appearing at two locations are coupled.

Figures 20 through 24 show the cross spectrum and phase differences of each long-wave between Monterey and Santa Cruz. Both raw spectra were smoothed over five frequency band widths. However, considerable difficulties arose when attempting to smooth the phase relationships.

The process of smoothing over band widths assumes that the data is the result of a stationary random process. The long-waves analyzed in this study may be viewed as deterministic functions and therefore what is considered in the record is a number of deterministic functions



superimposed on so called "white noise". The ideal way to smooth the phase information would be to smooth over ensembles, however, this could not be done because the long-waves are, in general, a transient phenomenon and usually not a stationary process. The Fast Fourier Transform (FFT) method of calculating the power spectrum is relatively new and, while the raw phase information is considered correct, the correct method to smooth relationships when using the FFT have not been completely worked out [Enochson and Otmes, 1968].

In this study the raw phase information was calculated utilizing the cross spectra as a weight function such that

$$\epsilon = \frac{\sum_i \epsilon_i \Phi_i}{\sum_i \Phi_i} \quad i = 1,2,3,4,5$$

where

$\epsilon$  = smooth phase difference

$\epsilon_i$  = raw phase

$\Phi_i$  = raw cross power.

This method was adopted after investigating several alternatives. The calculation resulted in those raw phase estimates associated with a high cross power to be dominant in the smoothing process. Those relationships are shown in Figures 20 through 24 and summarized in Table IX for selected periods. There are reasons to doubt the accuracy of calculating the smoothed phase using FFT in this manner which circumvents computing the correlation function. Hence, the smoothed phase calculation must be viewed with reservations.

Table VIII is a summary of significant cross spectral peaks for selected long-waves which recur throughout the five days analyzed. Figures 20 through 24 show the raw and smoothed cross spectra for the



PERIOD (minutes)	DATE				
	6 Nov 69	3 Dec 69	6 Dec 69	14 Jan 70	20 Jan 70
51.8	x		x	x	
42.5	x				
36.1		x			x
27.7			x	x	x
22.5					
18.9			x	x	
16.3	x				
13.6		x			
10.1	x	x	x		
9.3			x		

TABLE VIII

Summary of Significant Cross Spectral Peaks in Monterey Bay





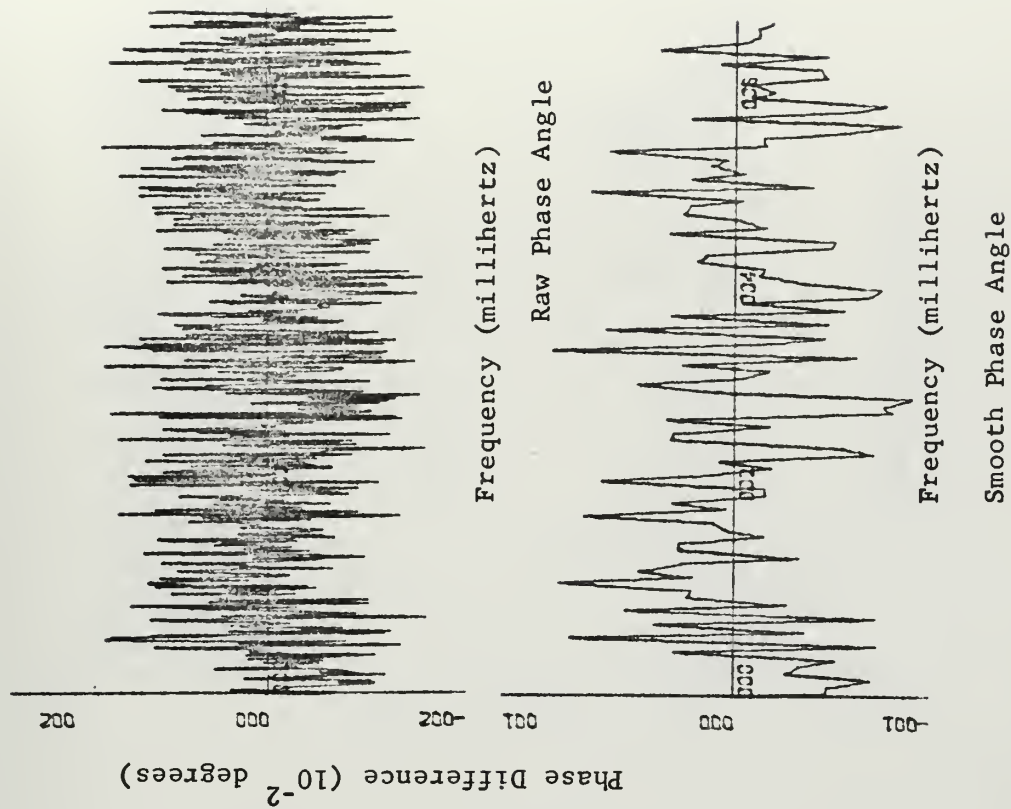
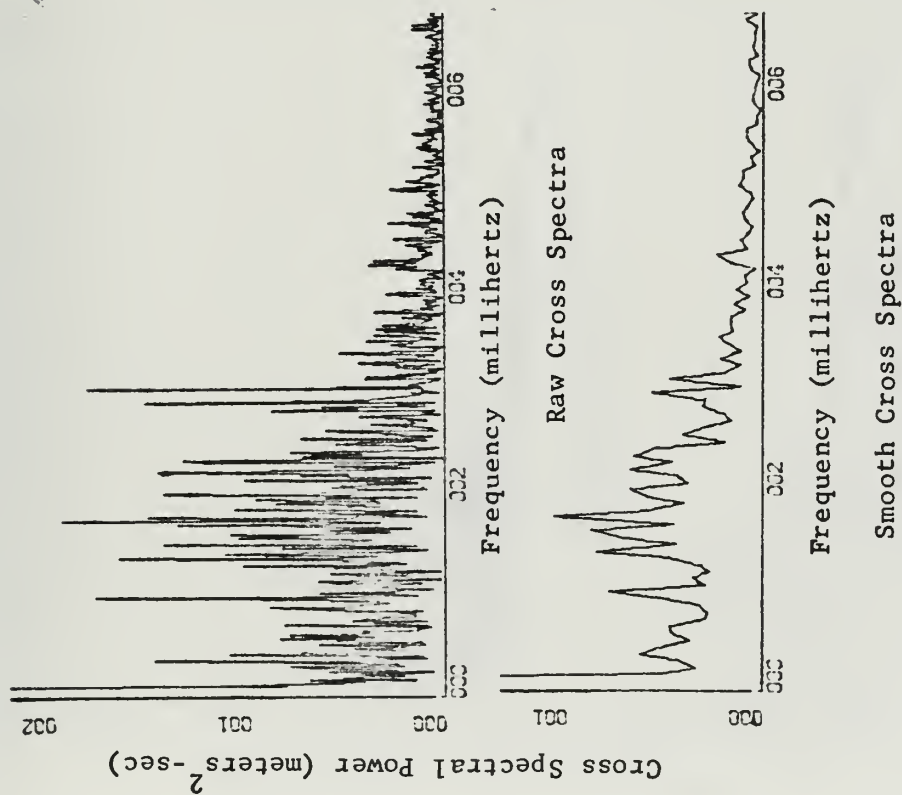


Figure 20

Cross Spectra and Phase Relationships, Monterey/Santa Cruz, 6 Nov 69



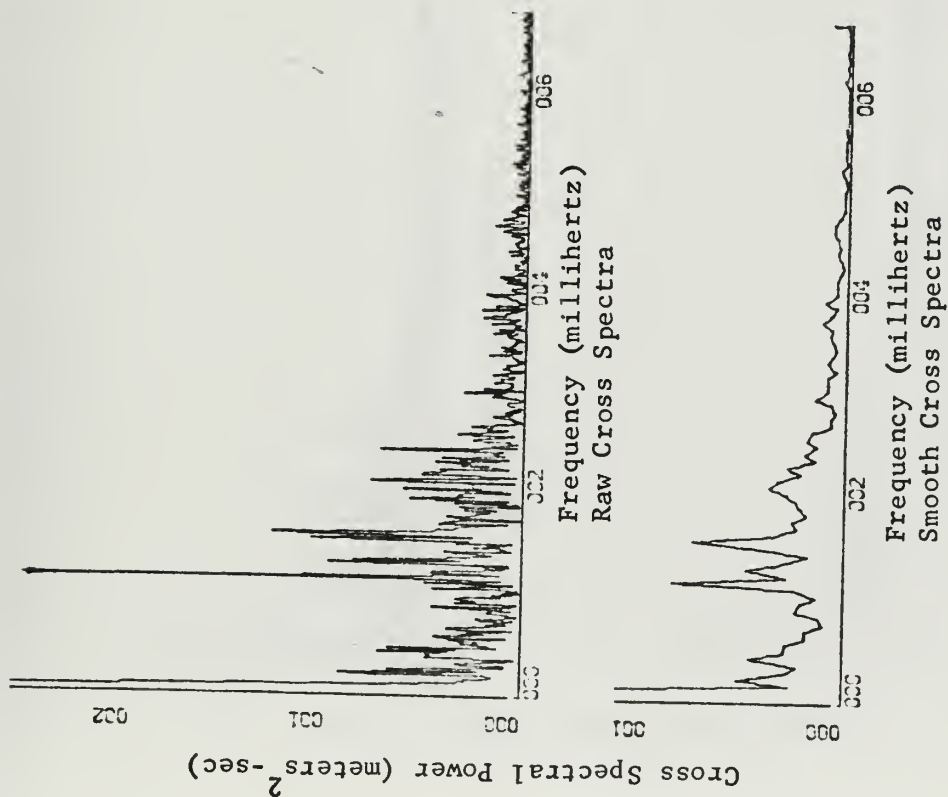
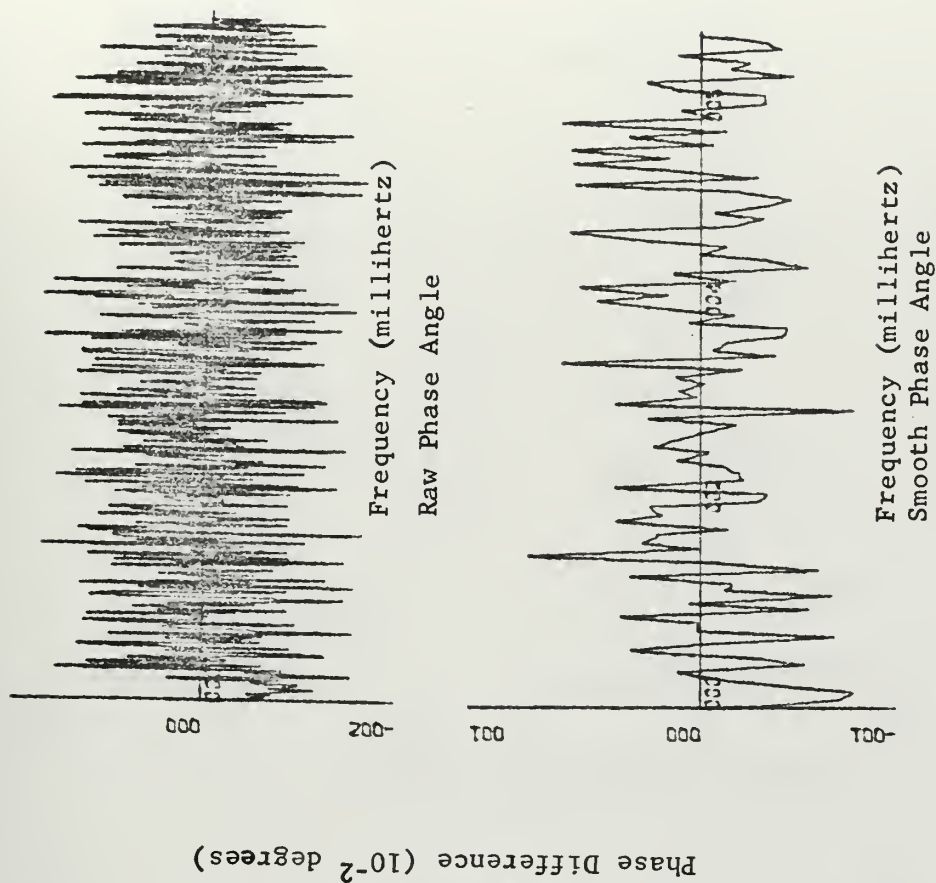


Figure 21

Cross Spectra and Phase Relationships, Monterey/Santa Cruz, 3 Dec 69





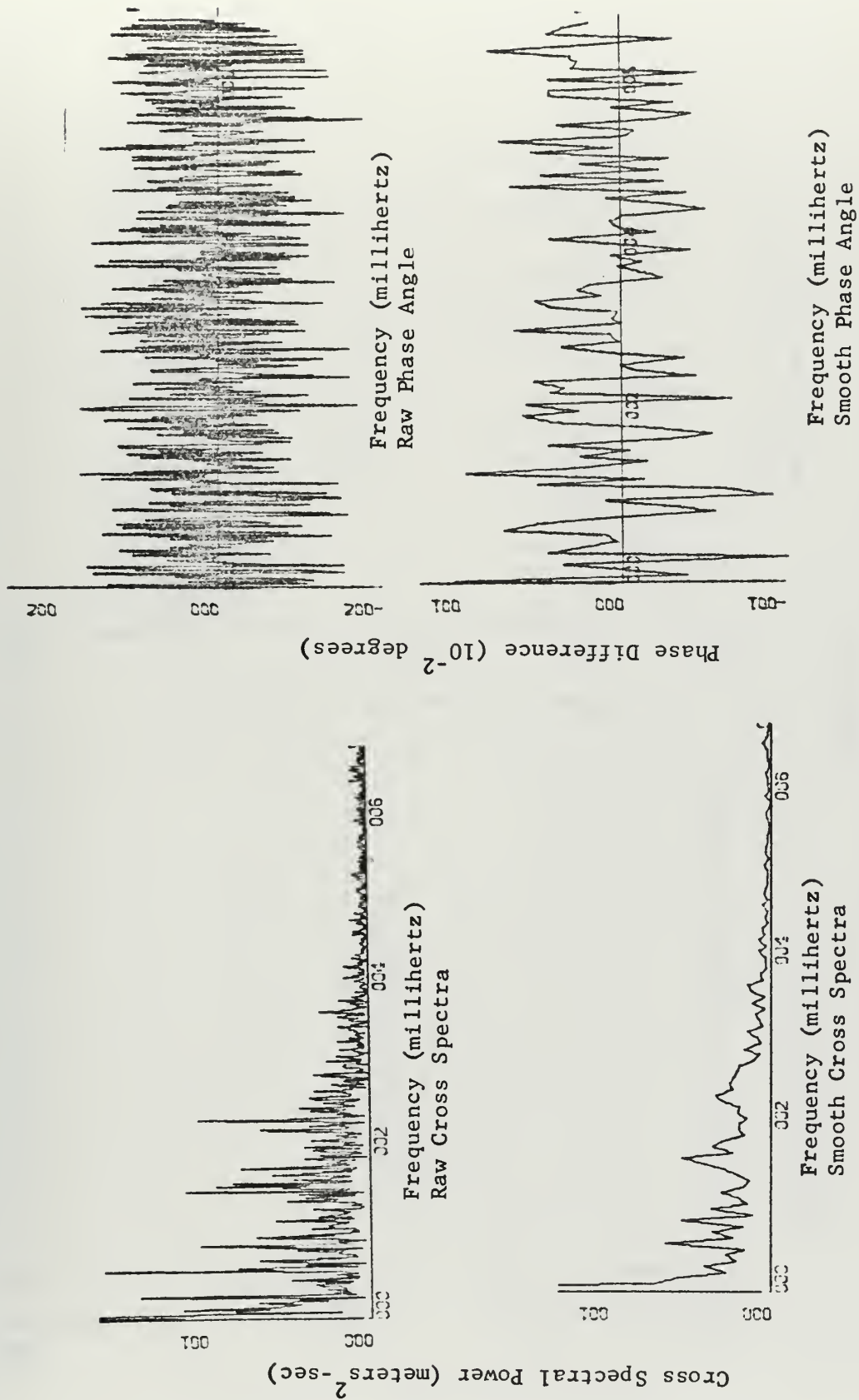


Figure 22

Cross Spectra and Phase Relationships, Monterey/Santa Cruz, 6 Dec 69



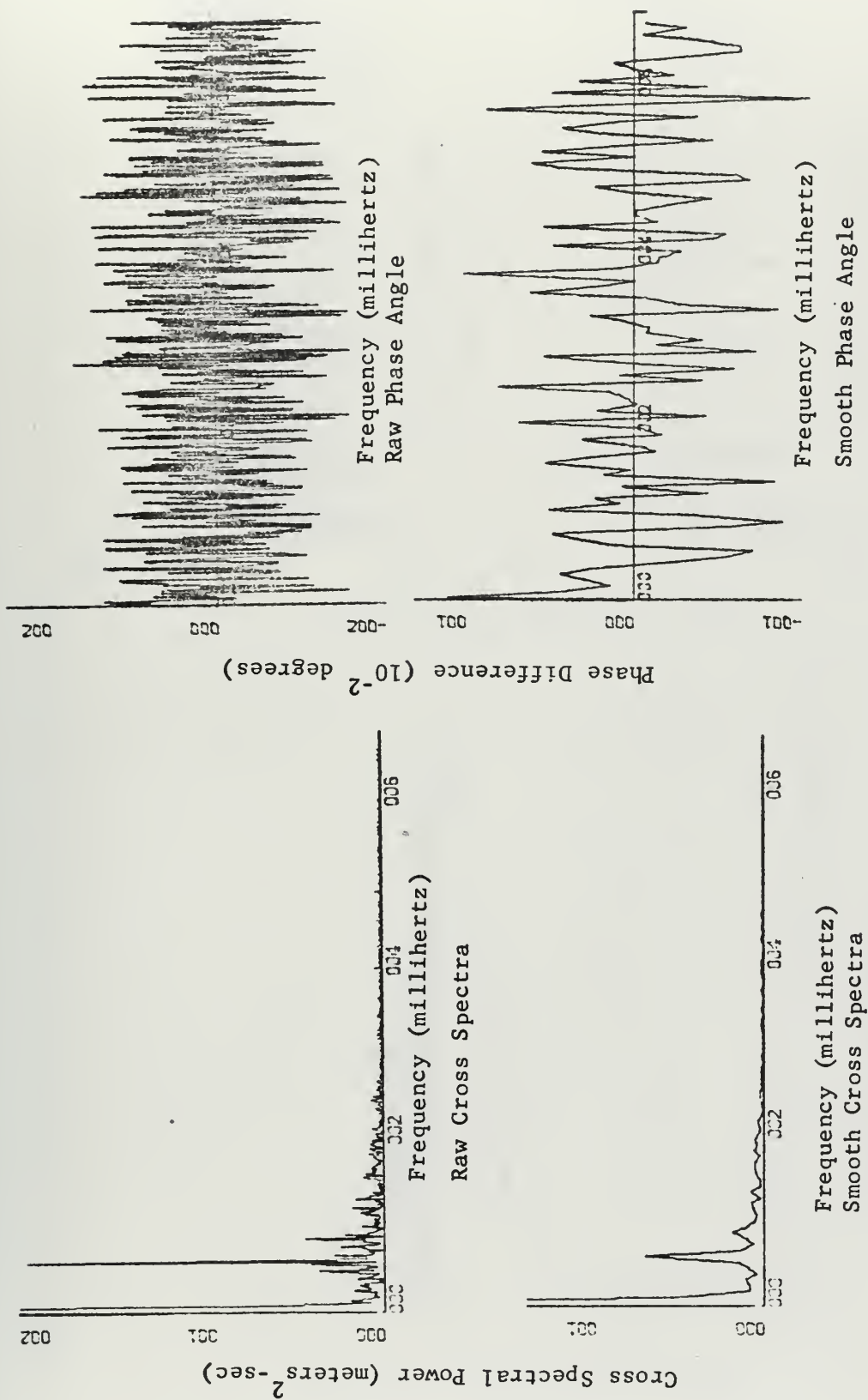


Figure 23

Cross Spectra and Phase Relationships, Monterey/Santa Cruz  
14 Jan 70







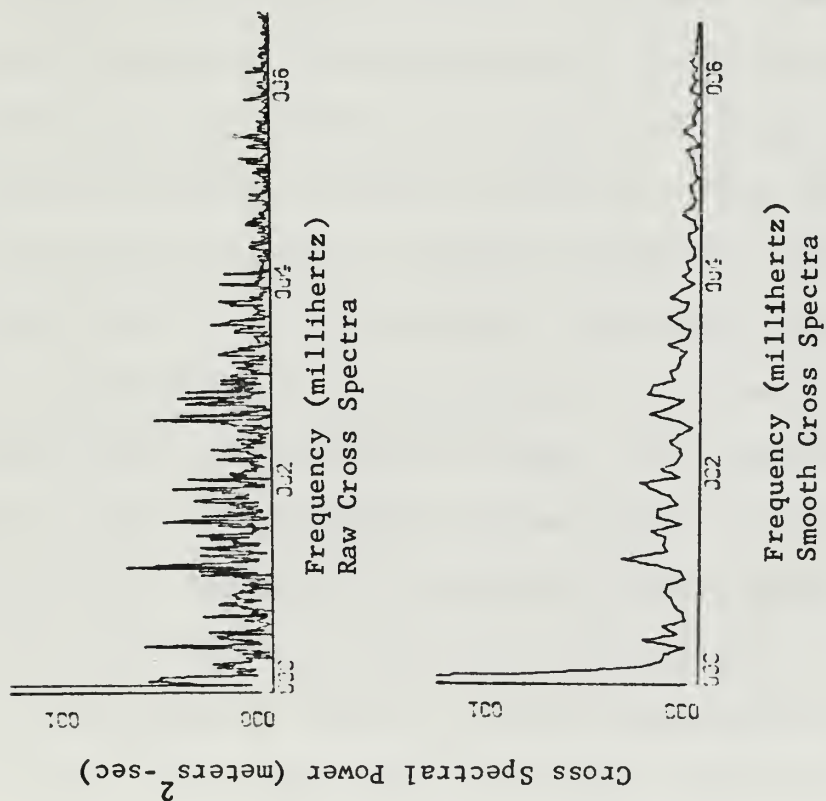
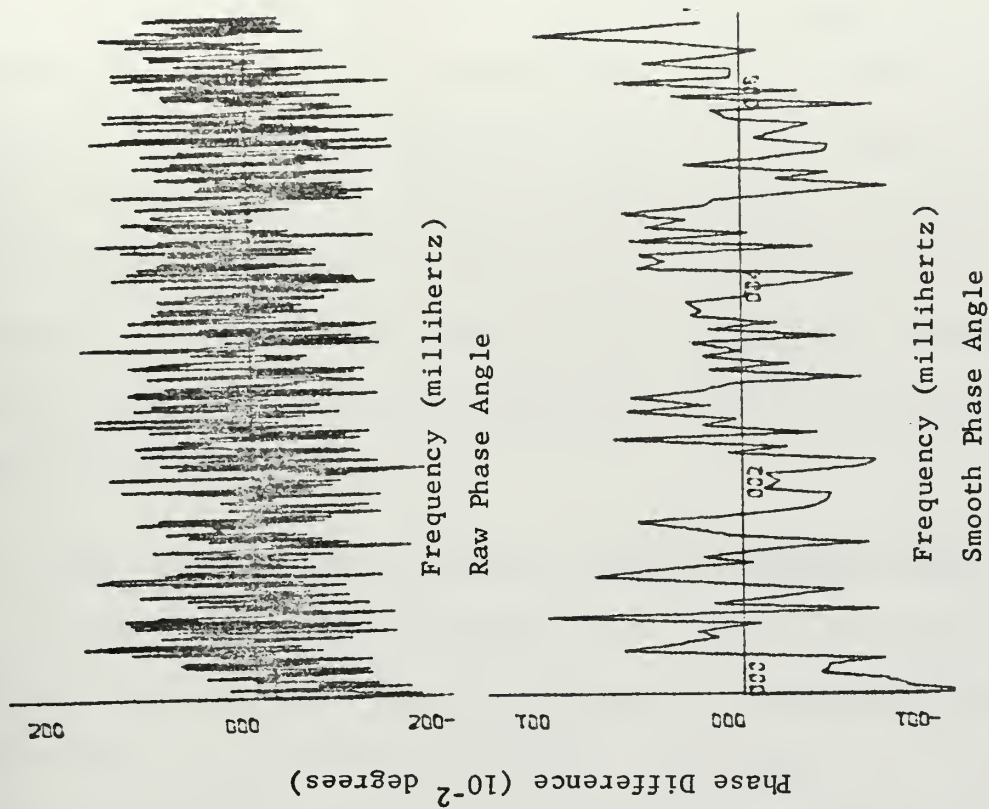


Figure 24

Cross Spectra and Phase Relationships, Monterey/Santa Cruz, 20 Jan 70



dates analyzed. As expected the cross spectra is dominated by the 27.7 minute shelf-wave and its associated harmonics. The cross spectral summary is in general agreement with Table VII, the summary of individual spectral peaks. The phase differences, between a particular wave at Santa Cruz and Monterey appear to be random, which implies the Monterey Canyon acts as a barrier dividing the bay into separate north-south basins which oscillate independently. This is in general agreement with Wilson [1965].

#### D. COMPARISON WITH WILSON'S THREE-DIMENSIONAL NUMERICAL MODEL

Wilson [1965] assumed a boundary nodal line for his solution of his three-dimensional model of the oscillating characteristics of Monterey Bay. This node is a line drawn from Pt. Santa Cruz to Pt. Pinos on the Monterey Peninsula. This assumption tends to be the greatest objection to Wilson's results, although an enclosed bay will usually have a node across the seaward opening for particular modes of oscillation. The oscillating characteristics are depicted in Figures 25 through 28 where Figure 25(a) gives the general bathymetry of the bay. These depths were used as grid points in order to generate the numerical solution. The other figures show the increasing complex longitudinal modes of oscillation. The contour lines are the amplitudes of the water level normalized to the highest anti-node for the mode. Each complex mode of oscillation is drawn in a simplified version to the left of the figure. The nodal line assumption is particularly constraining to the longitudinal mode of oscillation which can be seen in Figure 25 where mode 1 (i.e., the fundamental) tends to oscillate transversely due to the nodal constraint. The lowest modes would be most affected by the nodal assumption and would have less constraint on higher modes.



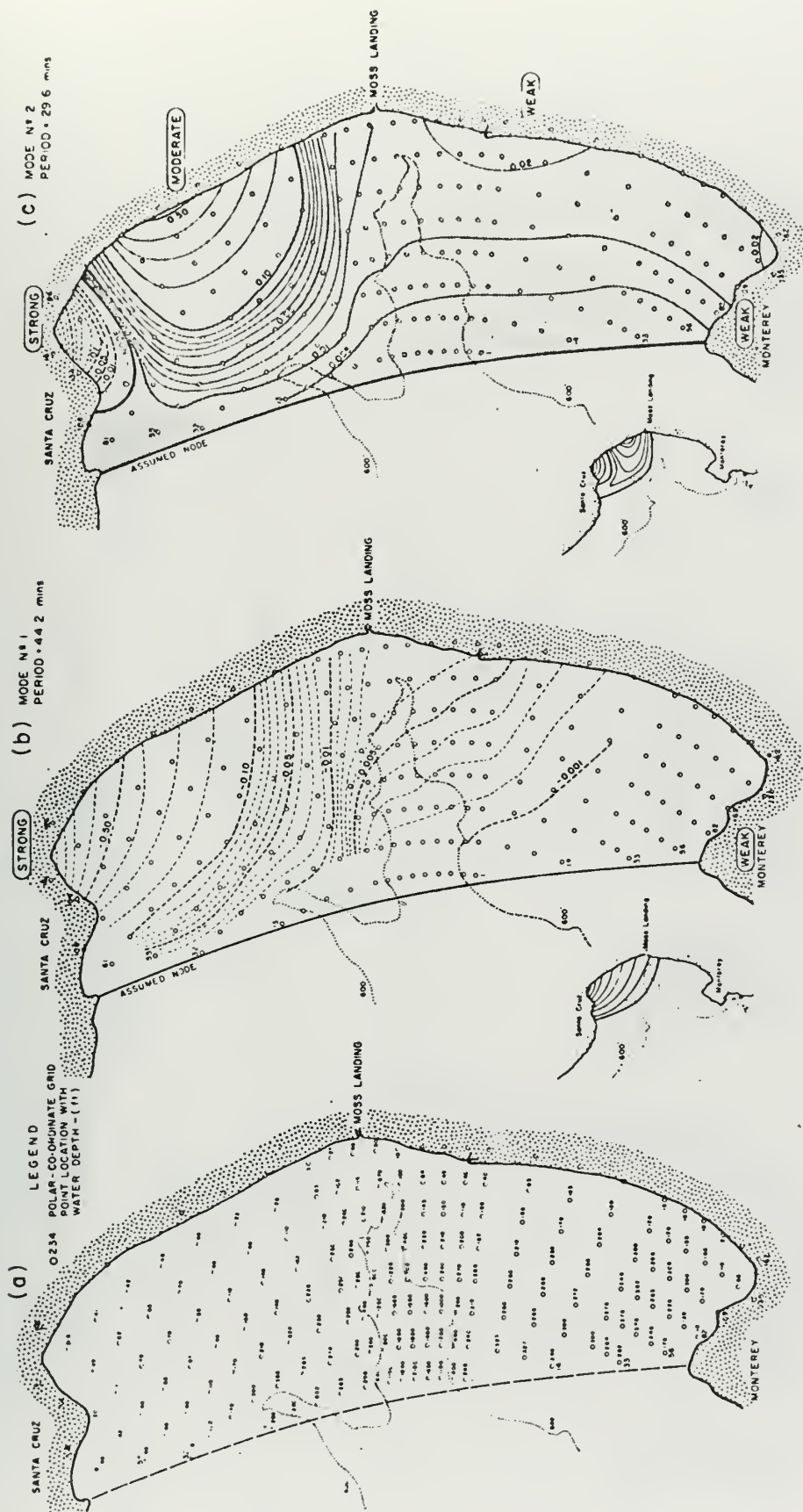


Figure 25

Numerical Calculations of Modes of Oscillation of Monterey Bay (from Wilson, 1965)











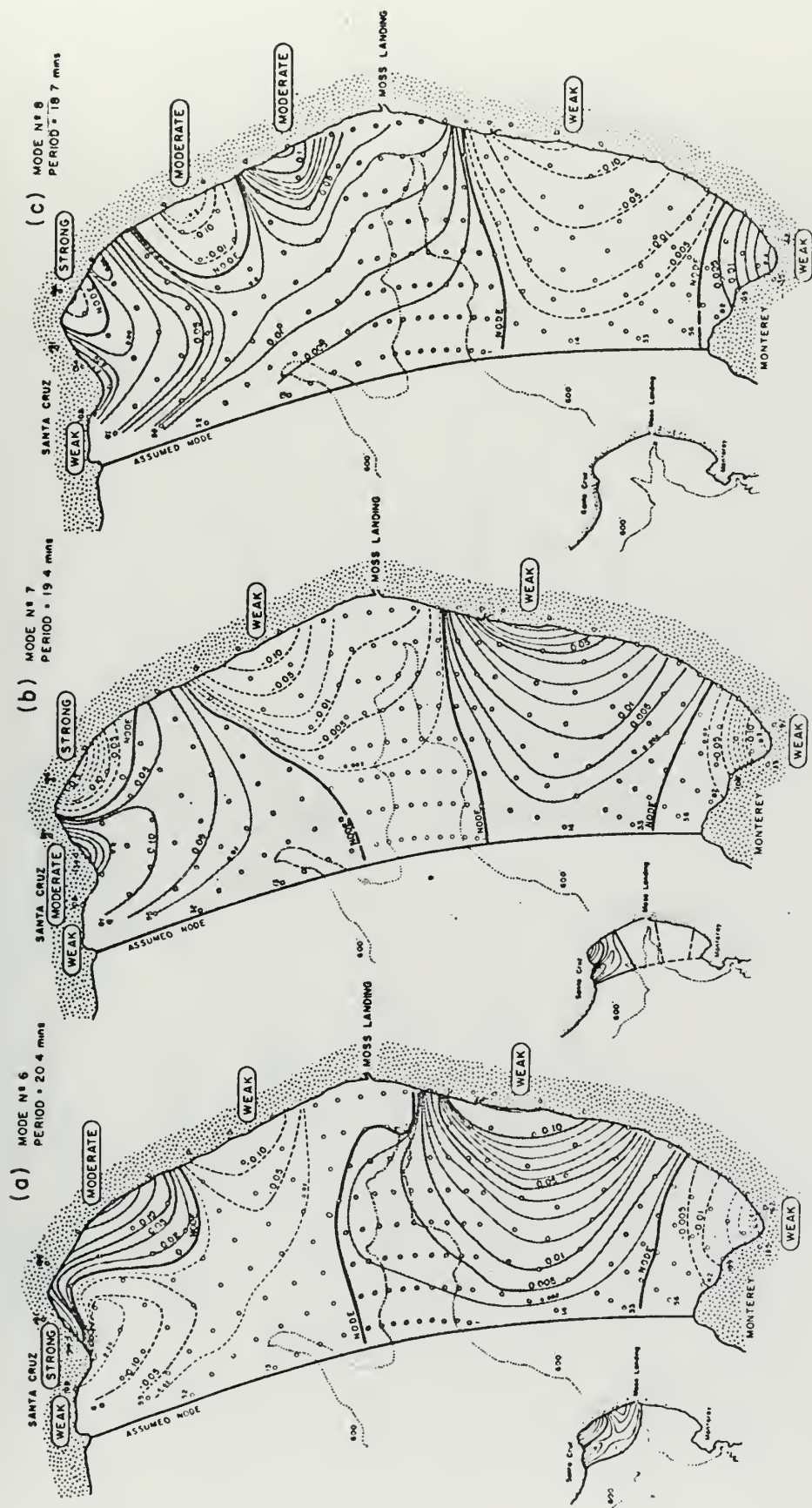


Figure 27

Numerical Calculations of Modes of Oscillation of Monterey Bay (from Wilson, 1965)



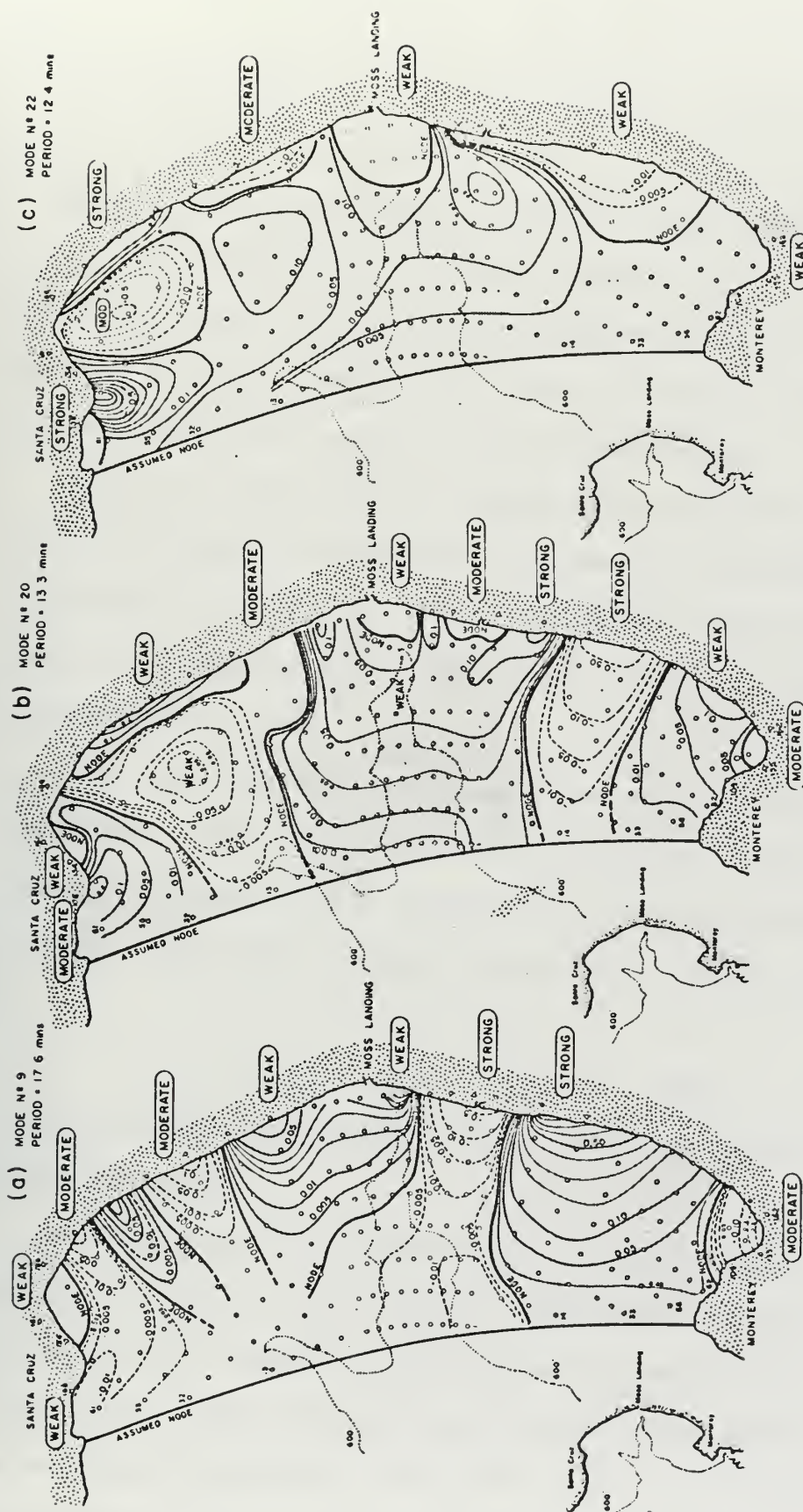


Figure 28

Numerical Calculations of Modes of Oscillation of Monterey Bay (from Wilson, 1965)



The periods of oscillation of the longitudinal mode were calculated as,

$$T_n = 44.2, 29.6, 28.2, 23.3, 21.6, 20.4, \\ 19.4, 18.7, 17.6, \dots \text{ mins.}$$

Mode 1 has a fundamental period of 44.2 minutes and indicates strong oscillation at Santa Cruz and weak at Monterey. Mode 2 is similar to mode 1 while mode 3 gives the only strong oscillation at Monterey and weak oscillations at Santa Cruz. Mode 4 is similar to modes 1 and 2, but is somewhat more complex as are successive modes.

Wilson does state that his assumed node may very well be incorrect and possible should be moved further seaward. The boundary conditions of the model would then be changed, resulting in new oscillating periods and characteristics. This would result in a lengthening of the fundamental period.

Wilson also computed two-dimensional modes of oscillation of Monterey Bay. This is the transverse oscillation of the east-west extremities bounded at Moss Landing on the east and the assumed node on the west. The sequence of modal periods of oscillation were calculated as

$$T_n = 32.3, 14.3, 9.5, 7.0, \dots \text{ mins.}$$

The profiles of water-surface elevation for the transverse mode are shown in Figure 29.

Considering both the longitudinal and transverse modes of oscillation, Wilson notes that decreased solution confidence is placed on each successive harmonic, however, the highest mode in which he has confidence is not stated. This increasing error in successive modal solutions can be decreased by choosing a finer grid. Wilson's model probably is most valid for modes greater than mode 2 and less than mode 7.





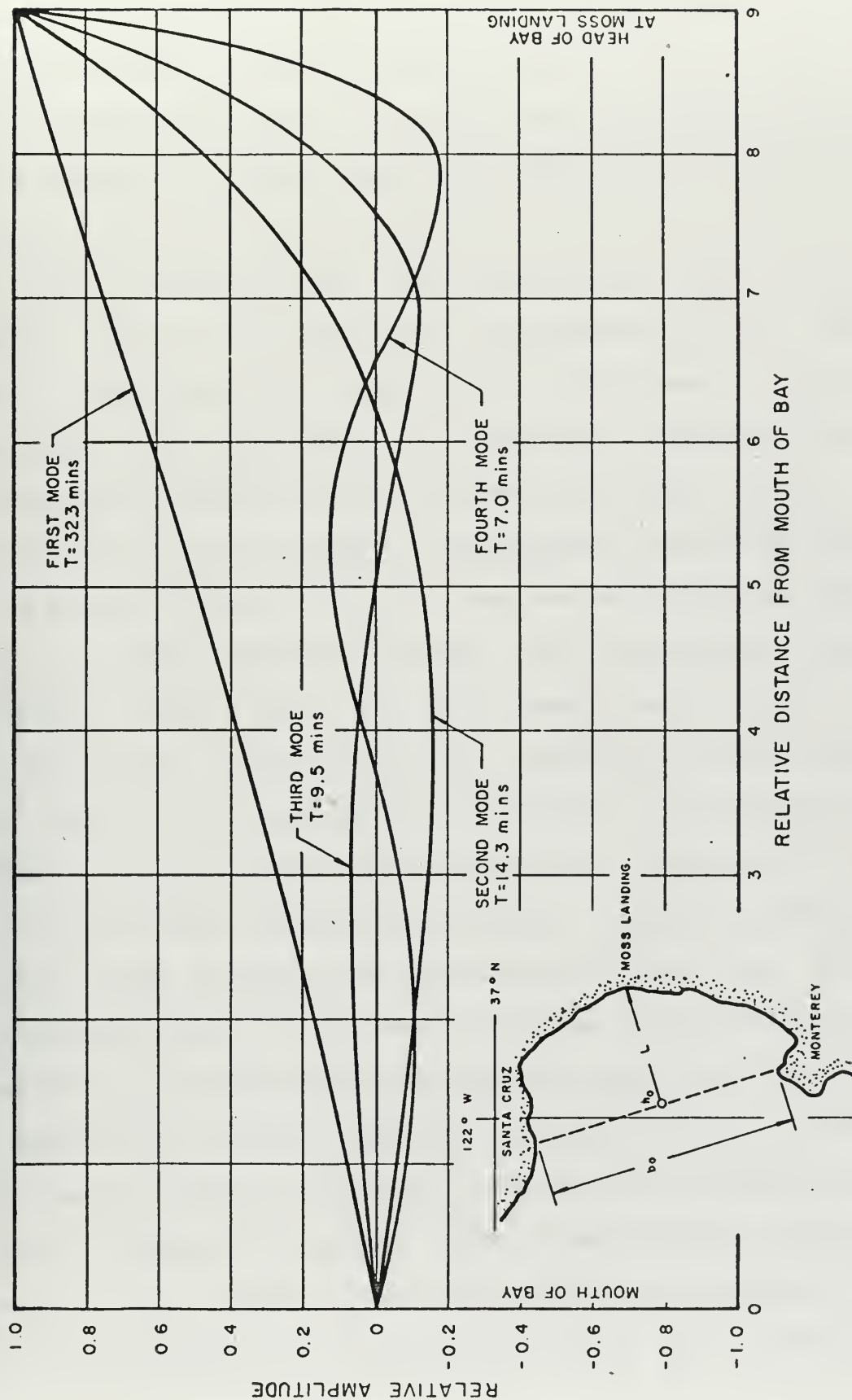


Figure 29  
Profiles of Water-Surface Elevation Along Axis of Bay for Four Lowest Modes of Transverse Oscillation in Monterey Bay (from Wilson, 1965)





As stated earlier, Wilson concluded that the deep Monterey Canyon does exert a profound influence on Monterey Bay oscillations as it effectively sets a barrier to free oscillations, between the north and south portions of the bay. The result is that wave activity acting in both locations would remain largely uncoupled and would give rise to a random phase.

Table IX summarizes some of the results of this study with that of Wilson's model. It is apparent that the long-wave activity at Santa Cruz is more intense than that at Monterey. This tends to support the conclusion that the two basins act independently. Robinson [1965] found similar magnitudes at both stations and concluded that the oscillations were not uncoupled. The fundamental longitudinal mode of 51.8 minutes was found to be weak at Monterey and stronger at Santa Cruz thus supporting Wilson's findings. The 20.5 minute wave corresponding to Wilson's mode 6 is in general weak at both stations. This is in agreement with Wilson's results. The greatest contrast between the results of this study and that of Wilson's is the effect and identification of the 27.3 minute wave, evaluated as a shelf-wave here, but as the third mode of oscillation by Wilson. This wave, according to Wilson, should be strong at Monterey and weak at Santa Cruz. In general, this was not found to be the case. The ratios of energy-density at Santa Cruz to energy-density at Monterey is greater than 1 on 6 November, 3 December and 6 December indicating the opposite of Wilson's findings. The last two periods studied show a ratio of .94 on 14 January 1970 and .49 on 20 January 1970, neither of which support Wilson's calculated results. It is considered that the 27.3 minute wave predicted by Wilson is in fact present during long-wave oscillations, but is evaluated as a



		POWER SPECTRA M <sup>2</sup> -SEC			CROSS SPECTRA		WILSON NUMERICAL MODEL			
Date	Period (min)	Monterey	Santa Cruz	Ratio $\left(\frac{SC}{M}\right)$	M <sup>2</sup> -sec	Phase Diff.	Monterey	Santa Cruz	Period	Mode #
6 Nov	51.8	.482	.522	1.04	.438	-32.4	weak	strong	44.2	1
	36.1	.835	.408	.50	.504	30.2	(Fundamental	Transverse	Mode)	
	27.7	.488	.582	1.19	.461	83.6	strong	weak	28.2	3
	20.5	.237	.534	2.26	.283	-71.0	weak	weak	20.4	6
	16.3	.647	1.382	2.15	.783	24.3	moderate	weak	17.6	9
	10.1	.879	.875	.99	.871	-14.1	(Transverse	Mode 3)		
3 Dec	51.8	.121	.927	7.25	.281	- 5.6	weak	strong	44.2	1
	36.1	.246	1.501	6.10	.475	-52.6	(Fundamental	Transverse	Mode)	
	27.7	.105	1.117	10.75	.296	35.7	strong	weak	28.2	3
	20.5	.080	.521	6.50	.109	1.01	weak	weak	20.4	6
	16.3	.093	.616	6.60	.219	-54.2	moderate	weak	17.6	9
	10.1	.712	1.423	2.00	.777	1.10	(Transverse	Mode 3)		
6 Dec	51.8	.226	.790	3.49	.400	-101.2	weak	strong	44.2	1
	36.1	.901	.246	.27	.278	21.1	(Fundamental	Transverse	Mode)	
	27.7	.370	1.177	2.89	.645	9.47	strong	weak	28.2	3
	20.5	.106	.325	3.06	.171	14.9	weak	weak	20.4	6
	16.3	.354	.413	1.17	.367	9.60	moderate	weak	17.6	9
	10.1	.396	.958	2.41	.547	-4.67	(Transverse	Mode 3)		
14 Jan	51.8	.094	.170	1.81	.109	43.8	weak	strong	44.2	1
	36.1	.125	.115	.94	.099	-20.1	(Fundamental	Transverse	Mode)	
	27.7	1.612	1.510	.94	.701	-70.2	strong	weak	28.2	3
	20.5	.098	.192	1.92	.105	48.5	weak	weak	20.4	6
	16.3	.035	.139	3.96	.056	-36.3	moderate	weak	17.6	9
	10.1	.023	.324	14.10	.056	52.2	(Transverse	Mode 3)		



20 Jan	51.8	.081	.586	7.25	.170	-43.0	weak (Fundamental strong weak moderate (Transverse Mode 3)	strong Transverse weak weak weak Mode 3)	44.2	1
	36.1	.342	.366	1.03	.281	60.7				3
	27.7	.350	.172	.49	.188	13.7				6
	20.5	.057	.520	9.12	.125	99.3				9
	16.3	.116	.468	4.01	.162	-22.6				
	10.1	.098	.256	2.62	.095	4.0				

TABLE IX

Comparison of Results with Wilson's Numerical Model



shelf-wave vice a harmonic of the fundamental frequency of oscillation of the bay. Most of the other periods predicted by Wilson were found to exist. However, it must be remembered that a discrete Fourier analysis was performed and the data smoothed. This has the result of "forcing" some energy into discrete band widths allowing only certain periods to appear in the spectrum.

As a result of this comparison the present study tends to verify many of Wilson's conclusions. The observations that wave amplitudes are generally higher at Santa Cruz and that the phase relationships tend to be random lead to the conclusion that oscillations of Monterey Bay at Santa Cruz and Monterey tend to operate independently of each other due to the impedance barrier formed by Monterey Canyon.





## V. ERROR ANALYSIS

Errors encountered during the course of the study were basically of three types:

- (1) Time errors can be introduced when recording raw data on an analog trace.
- (2) Errors may be generated in matching the Santa Cruz raw data to Monterey data.
- (3) The effect of the numerical methods of calculating the power-spectrum and the cross spectrum can lead to misinterpretation of results.

Each of these effects are examined below in order to determine, in a qualitative sense, their effect on the calculated results.

### Time Induced Errors

The tide gages at Santa Cruz and Monterey are both mechanical devices which must be wound at required intervals. Both were maintained within the required periods. The sampling interval of 34.495 seconds for Monterey and 6.00 seconds for Santa Cruz were computed by inches/time relationships between two successive time checks on each record for each period analyzed. The times at Santa Cruz were recorded from radio station WWV (Naval Observatory Time) and are correct to the second. The times at Monterey were obtained from the telephone company and are accurate within  $\pm 5$  seconds. Time checks were maintained daily for the entire period. The recording speed at Monterey was found to be 1.03  $\pm$  .01 inches/hour and 6.00  $\pm$  .02 inches/hour for Santa Cruz. Synchronized records are critical to phase computations. However, the resulting error in obtaining the raw data was considered insignificant.



### Errors Generated in Aligning Records

The Santa Cruz record was aligned with the Monterey record by converting the coordinate system from curvilinear coordinates to rectilinear coordinates. Linear interpolation was then used to match data points.

The Santa Cruz data yielded a data point every six seconds giving high resolution in digitizing. The original 12,000 plus data points were reduced to 2048 points. Reduction and conversion of data in this manner resulted in very minor raw data errors.

### Numerical Calculation Methods

The effect of approximating a continuous tidal trace with a step-function and transforming the step-function by numerical methods through use of a digital computer has several effects on the final analyses, two of which could result in misleading results of interpretation.

When a continuous curve is approximated by a step function, a secondary wave form of high frequency energy is superimposed on the recorded wave form. To evaluate this possible error the sampling rate was reduced and records were analyzed and checked for aliasing. It was found the aliased power was quite small and in no case greater than 10%.

The second source of error is the effect of transforming the record into discrete values of energy-density as a function of discrete periods. Thus, the energy is forced into certain band widths and centered about the middle frequency. As a result of smoothing over five frequencies, the frequency interval was .00007 cps ranging in periods from 397.4 minutes to 1.16 minutes. Comparison of raw and smooth power and cross spectra show that the raw spectra was not smeared due to smoothing the original 1025 spectral estimates as "spikes" in the raw spectra were still present after the smoothing process was completed.



## VI. SUMMARY

Power spectra analyses and cross spectral analyses of wave records in Monterey Bay indicate the presence of persistent unique periods with the phase differences between Monterey and Santa Cruz tending to be random. Coupled with this and the fact that wave energy at Santa Cruz is in general greater than that at Monterey, it is concluded that the Monterey Submarine Canyon has a profound effect on seiching motions within the bay. This effect appears to divide the Monterey Bay into north and south basins where oscillations occur essentially independent of each other.



## APPENDIX A

### DIGITIZING GRAPHICAL ANALOG RECORDS USING THE CALMA COMPANY MODEL 480

The Calma digitizer reduces analog graphical data to digital magnetic tape for computer processing and analysis. It consists of a freestanding tracing bed and a separate electronics/recorder module. The tracing bed is motor-driven for adjustment to the most comfortable digitizing position.

To digitize analog graphical data directly on magnetic tape, the operator traces the analog curve with a moveable stylus/carriage assembly. As the curve is traced, movements of the stylus in either the X or Y direction causes pulsed to be generated by the direct digital pickoff mechanism. Incremental motions are recorded as characters on magnetic tape, leaving the task of summing the increments for whole-value coordinates to the final processing computer. This method of digitizing is both faster and more accurate than template methods.

The digitizer reads and records data in the X and Y direction, with a sampling interval which can be set to 0.01, 0.02, 0.04, 0.08, or 0.15 inches. The maximum absolute sampling error for the machine is 0.012 inches. The output is external BCD, stored on 556 bpi, 7 channel, tape. The tape can be made compatible with the IBM/360 system, however, it is easier to use the CDC/6500 computer.

A FORTRAN IV program wirtten for the CDC/6500 can be utilized for interpreting the digitized record.

The program accomplished the following:

- (1) Converts the data from external BCD to display code.
- (2) Arranges the data in column matrices of 80 character length.





(3) Interrogated each character in the matrix to determine if the character is a

(a) Point flag - a designator to label a specific data point of interest

(b) Plus or minus travel in the X-direction.

(c) Plus or minus travel in the Y-direction.

(4) Sums the Y direction travel.

(5) Records incremental travel in X direction (plus or minus).

This allows for negative travel while digitizing. Negative travel is common if digitizing a curvilinear trace or if variations in data are closely spaced.

(6) Prints the Y value  $V(N)$ , for each designated increment in the X direction.

(7) Prints the X increment value  $U(N)$ , showing the incremental travel (plus or minus) for which the Y value was obtained.

(8) Punch cards for  $V(N)$  and  $U(N)$  which are used as inputs for other computer programs as desired.

Headings may be entered on the tape with keyboard control, but CONVERT is not designed to read tape headings nor does the program have the capability to search for a particular set of data on the tape. Consequently, all data on the tape will be analyzed each time CONVERT is used.

The following sequence of operations can be used as a guide for the digitizing procedure:

(1) Mount chart on tracing bed so that horizontal axis of chart is aligned with X axis of digitizer.

(2) Turn system power on.



- (3) Load tape.
- (4) Initiate "Load Forward" on tape module.
- (5) Press RECORD ERROR. (If recorder is properly loaded, RECORD ERROR indicator will go out when push button is pressed,... digitizer will automatically shift to KEYBOARD MODE.)
- (6) Enter required identifiers from keyboard if necessary.  
(Not required with CONVERT program.)
- (7) Position stylus at beginning of curve. Lock Y-axis, and check alignment.
- (8) Initiate TRACER Mode, de-activate SKIP (SKIP button out, light off), and unlock stylus.
- (9) FLAG the tape once.
- (10) Trace the record as desired. The first point which will not be computed, or printed, will be  $X = 0$ ,  $Y = 0$ . The speed at which the stylus can be moved without destroying data accumulated is well in excess of the speed normally used to digitize the record. However, if exceeded, the RECORD ERROR will light and record will be deleted.
- (11) When tracing is complete, lock the stylus, initiate FLAG PEDAL twice and IRG button once.
- (12) Press rewind button on tape module and remove tape.

The user of the Calma digitizer can reap the benefits of the instrument's speed and accuracy only if the digitizing operator understands and applies the system's basic rules of operation. It is strongly suggested that new digitizing operators read, study and understand the textual instructions (CALMA MODEL 480 DIGITIZER, Instruction and Maintenance Manual) before attempting to operate the digitizer.



- (13) The magnetic tape from the digitizing process is used as INPUT to program CONVERT. The approximate number of words on the tape should be estimated so that CONVERT may be dimensioned accordingly.

In addition, if the tape contains any parity errors, or was not deguassed properly prior to usage, the program will only interpret a portion of the data or possibly not run at all. These problems have been encountered in the past.

- (14) The program CONVERT is included and is self-explanatory.

Output consists of (a) sampling rate, (b) length of record (sec), (c) amplitude values (inch)  $V(N)$ , and (d) incremental travel in X-direction  $U(N)$ . These may be summed in another program to determine proper time or spatial frame.

- (15) It must be remembered that CONVERT is FORTRAN IV written for the CDC/6500 located at FNWC Monterey, and must be adapted for other computers.



```

PROGRAM CONVERT(INPUT,OUTPUT,PUNCH)
C THIS PROGRAM CONVERTS THE DIGITIZED RECORD OF THE
C MONTEREY TIDE GAUGE FROM EXTERNAL BCD TO AMPLITUDE
C VALUES FOR A CONSTANT SAMPLING RATE DELT, AVGX IS THE
C AVERAGE DISTANCE THE RECORDING DRUM ADVANCES PER ONE
C HOUR REAL TIME, I.E, THE DATA SAMPLING RATE.
C REFERENCE-CALMA CO, MODEL 303 DIGITIZER INSTRUCTION
C MANUAL. DIMENSION STATEMENT MAY BE MODIFIED TO
C COMPUTE AS MANY DATA POINTS AS REQUIRED.
C DIMENSION U(17000),V(17000),N(80),IBUFF(17000),NK(80)
C PROGRAM READS L SETS OF DATA FROM TAPE
  READ 97,L
  97 FORMAT(I2)
  DO 116 JJ=1,L
  READ 98,AVGX
  98 FORMAT(F10.3)
  C COMPUTE SAMPLING RATE (SECONDS)
  DELT=36.0/AVGX
  PRINT 99,DELT
  99 FORMAT(1H0,2X,25H SAMPLING INTERVAL EQUALS,1X,E15.7,1X
  1,7HSECONDS)
  PRINT 700,JJ
  700 FORMAT(1H0,9HJJ EQUALS, 2X,I2)
  C ZERO FILL AMPLITUDE ARRAYS
  DO 100 I=1,17000
  U(I)=0.0
  V(I)=0.0
  100 CONTINUE
  C ZEROIZE INPUT/OUTPUT BUFFER
  101 DO 202 I=1,17000
  IBUFF(I)=0
  202 CONTINUE
  COUNTX=0.0
  COUNTY=0.0
  NUM=17000
  C COUNT NUMBER OF DATA POINTS ACCUMULATED
  M=1
  K8=0
  C CALL SUBROUTINE LIOF TO READ INPUT TAPE WHERE NPAR IS
  C A PARITY CHECK,AND NEOF, CHECKS FOR END OF FILE
  CALL LIOF(5LRBCD1,IBUFF,NUM,NPAR,NEOF)
  IF(NEOF) 602,200
  200 K=-7
  KF=0
  201 K=K+8
  KA=K+8
  KC=0
  DO 102 KB=K,KA
  KC=KC+1
  NK(KC)=IBUFF(KB)
  IF(NK(KC).EQ.0) KF=1
  102 CONTINUE
  C DECODE TAPE 80 WORDS AT A TIME
  DECODE(80,103,NK) (N(I),I=1,80)
  103 FORMAT(80R1)
  C INTERPRET BCD ON TAPE AND COMPUTE AMPLITUDES
  DO 104 I9=1,80
  IF(N(I9).EQ.508) GO TO 106
  IF(N(I9).EQ.558) GO TO 107
  IF(N(I9).EQ.348) GO TO 108
  IF(N(I9).EQ.478) GO TO 105
  C SYMBOL / (508) REPRESENTS AN INCREMENT TRAVEL IN THE
  C MINUS X OR Y DIRECTION BY THE DIGITIZER.
  C SYMBOL 0,(558),REPRESENTS A ZERO INCREMENT TRAVEL IN
  C THE X OR Y DIRECTION BY THE DIGITIZER.
  C SYMBOL 1,(348),REPRESENTS AN INCREMENT TRAVEL IN THE
  C POSITIVE X OR Y DIRECTION BY THE DIGITIZER.
  C SYMBOL *,(478), IS A FLAG INSERTED IN THE RECORD BY
  C THE PERSON DIGITIZING.
  GO TO 104
  105 PRINT 205,M,I9
  205 FORMAT(1H0,2I10)

```





```

C      THIS SECTION ALLOWS FIRST TEN WORDS OF TAPE TO BE
C      IDENTIFICATION DATA IF DESIRED.
      IF(M.GT.10) GO TO 113
      GO TO 104
C      COMPUTE NEGATIVE TRAVEL
106  RX=-0.01
      K8=K8+1
      GO TO 109
C      COMPUTE ZERO TRAVEL
107  RX=0.0
      K8=K8+1
      GO TO 109
C      COMPUTE POSITIVE TRAVEL
108  RX=0.01
      K8=K8+1
109  K3=K8/2
      K3=2*K3
      IF(K3.EQ.K8) GO TO 111
      COUNTX=COUNTX+RX
      IF(COUNTX.NE.0.0) GO TO 110
      GO TO 104
C      COMPUTE X INCREMENT TRAVEL
110  U(M)=COUNTX
      GO TO 104
111  COUNTY=COUNTY+RX
      IF(COUNTX.NE.0.0) GO TO 112
      GO TO 104
C      COMPUTE AMPLITUDE VALUE
112  V(M)=COUNTY
      COUNTX=0.0
      M=M+1
C      STOP IF DATA POINTS EXCEED ARRAY
      IF(M.GT.17000) GO TO 600
104  CONTINUE
      IF(KF.EQ.1) GO TO 113
      GO TO 201
C      TOTAL TIME OF THE RECORD EQUALS THE NUMBER OF DATA
C      POINTS TIMES THE SAMPLING INTERVAL, DELT.
      TIME=(M*DELT)/3600.0
      PRINT 115, TIME
115  FORMAT(1H,10X,28H TOTAL TIME OF RECORD EQUALS,2X,E15.
17,2X,7H HOURS,)
C      PRINT AND PUNCH AMPLITUDE VALUES
      PRINT 215,(V(I),I=1,M)
      PUNCH 213,(V(I),I=1,M)
C      PRINT AND PUNCH X INCREMENTS
      PRINT 215,(U(I),I=1,M)
      PUNCH 215,(U(I),I=1,M)
215  FORMAT(1H,10X,14F7.2)
213  FORMAT(14F5.2)
116  CONTINUE
      STOP
600  PRINT 601
601  FORMAT(1H1,20X,35H**** U AND V SPACE INADEQUATE *****)
602  STOP
      END

```



## APPENDIX B

### COMPUTATION OF FAST FOURIER TRANSFORM USING IBM/360 SUBROUTINE RHARM

This subroutine is a one-dimensional Fast Fourier Transform (FFT) analysis based on the Cooley and Tukey [1965] algorithm. The program is designed to analyze  $I$  data points where

$$I = 2^M \quad M = 3, 4, 5, \dots, 20.$$

The FFT not only greatly reduces the number of calculations from earlier analysis schemes, but also reduces the round-off errors in the Fourier coefficients. The computations and round-off errors are reduced by  $\log_2(I)/I$ .

The subroutine HARM is then called by RHARM to compute the complex coefficients:

$$A_K = \frac{1}{N} \sum_{j=0}^{N-1} (x_{2j} - ix_{2j+1}) \frac{2\pi i}{N} jK$$

$$K = 0, 1, 2, \dots, N-1.$$

Then for  $K = 1, 2, \dots, \frac{N}{2} - 1, \frac{N}{2}$  (with the bar denoting conjugation)

$$A'_K = \frac{1}{2} (\bar{A}_K + A_{N-K}), \quad A''_K = \frac{1}{2} (\bar{A}_{N-K} - A_K)$$

and,

$$C_K = \frac{1}{2} (A'_K + \bar{A}''_K e^{i(\frac{\pi}{2} - \frac{\pi}{N} K)})$$

$$\text{for } K = 1, 2, \dots, \frac{N}{2} - 1, \frac{N}{2}$$

$$C_{N-K} = \frac{1}{2} (\bar{A}'_K - A''_K e^{i(\frac{\pi}{N-K} - \frac{\pi}{2})})$$

$$\text{for } K = 1, 2, \dots, \frac{N}{2} - 1.$$



If we let  $C_0 = \frac{1}{2} (\text{Re}A_0 - \text{Im}A_0)$

$$C_N = \frac{1}{2} (\text{Re}A_0 + \text{Im}A_0)$$

We finally have:

$$\begin{aligned} A_0 &= 2 \text{ Re } (C_0), & B_K &= -2 \text{ Im } (C_K), \\ B_0 &= 0, & A_N &= 2 \text{ Re } (C_N), \\ A_K &= 2 \text{ Re } (C_K), & B_N &= 0. \end{aligned}$$

The Fourier coefficients  $A_0, B_0 = 0, A_i, B_i, \dots, A_N, B_N = 0$  are obtained for input  $X_j, j = 0, 1, 2, \dots, 2N-1$ , where  $N = I/2$ , for the following equation.

$$\begin{aligned} X_j &= \frac{1}{2} A_0 + \sum_{K=1}^{N-1} (A_K \cos (\pi j K / N) + B_K \sin (\pi j K / N) + \\ &\quad \frac{1}{2} A_N (-1)^j). \end{aligned}$$



## APPENDIX C

### LEAST SQUARES CURVE FITTING

The scheme used to accomplish the least squares fit for a sixth-degree polynomial is described below. Given a set of data pairs  $(X_i, Y_i)$  ( $i = 0, N$ ) which can be interpreted as measured values of the coordinates of the points on the graph of  $y = f(x)$ , assume that the unknown function  $f(x)$  can be approximated by a linear combination of suitably chosen functions,  $f_0(x), f_1(x), \dots, f_6(x)$  of the form  $F(x) = A_0 f_0(x) + A_1 f_1(x) + A_2 f_2(x) + A_3 f_3(x) + A_4 f_4(x) + A_5 f_5(x) + A_6 f_6(x)$  where the unknown coefficients  $A_0, A_1, \dots, A_6$  are independent parameters to be determined and the degree of least squares polynomial,  $M$  is such that  $M = 6 < N$ . The difference between the approximating function value  $F(x_j)$  and the corresponding data value  $Y_j$ , is called the residual  $r_j$  and is defined by the relation

$$r_j = F(X_i) - Y_i \quad (i = 0, N).$$

The function  $F(x)$  that best approximates the given set of data in a least squares sense is that linear combination  $A_0 f_0(x) + A_1 f_1(x) + \dots + A_6 f_6(x)$  of functions  $f_k(x)$  that produces the minimum value of the sum  $Q$  of the squared residuals where

$$Q = \sum_i r_i^2 \equiv \sum_i [F(X_i) - Y_i]^2.$$

For the case of curve-smoothing by polynomial least squares, we approximate the function  $y = f(x)$  over the range of data points  $(X_i, Y_i)$  ( $i = 0, N$ ). The parameters  $A_0, A_1, \dots, A_6$  are then determined such that





$$Q = \sum_i r_i^2 = \sum_i [P_6(X_i) - Y_i]^2$$

is a minimum. That is, a 6th degree polynomial curve is fitted to the data points in a least squares sense as defined earlier. The normal equations for the least-squares polynomial can be written as

$$\begin{bmatrix} N + 1 & \sum X_i & \sum X_i^2 & \dots & \sum X_i^6 \\ \sum X_i & \sum X_i^2 & \sum X_i^3 & \dots & \sum X_i^7 \\ \dots & \dots & \dots & \dots & \dots \\ \sum X_i^6 & \sum X_i^7 & \sum X_i^8 & \dots & \sum X_i^{12} \end{bmatrix} \begin{bmatrix} A_0 \\ A_1 \\ \dots \\ A_6 \end{bmatrix} = \begin{bmatrix} \sum Y_i \\ \sum X_i Y_i \\ \dots \\ \sum X_i^6 Y_i \end{bmatrix}$$

where:

$N$  = number of data pairs

$X_i$  = time increment (seconds)

$Y_i$  = amplitude values (inches)

The solution of the matrix is the sixth-degree polynomial,

$$P_6(x) = A_0 + A_1 X_1^1 + A_2 X_2^2 + A_3 X_3^3 + A_4 X_4^4 + A_5 X_5^5 + A_6 X_6^6$$

which is then subtracted from each data point to remove tidal effects, acting as a high-pass filter. Choosing a sample interval of 90 increments, 23 data pairs were utilized for the least squares fit.



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